1) $\lim_{x\to 1} \frac{4x^2-7x+3}{5x^2-8x+3} = \frac{(x-1)(4x-3)}{(x-1)(5x-3)} = \frac{(4\times1-3)}{(5\times1-3)} = \frac{1}{2}$

ب) با اسپرلر

ک) $(x-1)$ عامل صفر کننده

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2) $\lim_{x\to 0} \frac{|1-3x|-|3x+1|}{x}$

$\lim_{x\to 0^+} \frac{(1-3x)+(-3x-1)}{x} = \frac{-6x}{x} = -6$

$\lim_{x\to 0^-} = \frac{(1-3x)-(3x+1)}{x} = -6$

ب) صورت

ک) $(x-0)$ عامل صفر کننده

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3) $\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} \times \frac{\sqrt{x}+2}{\sqrt{x}+2} = \frac{(x-4)(\sqrt{x}+2)}{(x-4)} \to \sqrt{4}+2 = 4$

ک) $(x-4)$ عامل صفر کننده

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4) $\lim_{x\to 2} \frac{x-\sqrt{2x}}{2x^2-x-6}$ ← $(x-2)$ عامل صفر کننده

$= \frac{(x-\sqrt{2x})}{(x-2)(2x+3)} \cdot \frac{(x+\sqrt{2x})}{(x+\sqrt{2x})} = \frac{x^2-2x}{(x-2)(2x+3)(x+\sqrt{2x})}$

$\to \frac{(x)(x-2)}{(x-2)(2x+3)(x+\sqrt{2x})} \to \frac{2}{(7)\cdot(4)} = \frac{1}{14}$

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5) $\lim_{x\to 4} \frac{\sqrt{3x+4}-4}{\sqrt[3]{5x+7}-3} \times \frac{\sqrt{3x+4}+4}{\sqrt{3x+4}+4} \times \frac{\sqrt[3]{(5x+7)^2}+9+3\sqrt[3]{5x+7}}{\sqrt[3]{(5x+7)^2}+9+3\sqrt[3]{5x+7}}$

$\to \lim_{x\to 4} \frac{(3x+4-16)}{(5x+7-27)} \times \frac{(\sqrt[3]{(5x+7)^2}+9+3\sqrt[3]{5x+7})}{(\sqrt{3x+4}+4)} \to \frac{(3)(x-4)(\sqrt[3]{(5x+7)^2}+9+\cdots)}{5(x-4)(\sqrt{3x+4}+4)}$

$\to \frac{3\times(9+9+9)}{5\times(4+4)} = \frac{81}{40}$ ← $(x-4)$ عامل صفر کننده

6) $\lim_{x\to 1} \frac{1-\sqrt{x}}{2-\sqrt{5-x}} \times \frac{1+\sqrt{x}}{1+\sqrt{x}} \times \frac{2+\sqrt{5-x}}{2+\sqrt{5-x}} \to (x-1)$ صفر شود

(-1)

$= \frac{(1-x)}{1+\sqrt{x}} \cdot \frac{2+\sqrt{5-x}}{4-(5-x)} = (-1) \times \frac{2+2}{2} = -2$

$(x-1)$

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7) $\lim_{x\to 1} \frac{\sqrt[3]{x+\sqrt{x}} - 2}{\sqrt[3]{x}-1} \times \frac{\sqrt[3]{x^2}+1+\sqrt[3]{x}}{\sqrt[3]{x^2}+1+\sqrt[3]{x}} \times \frac{\sqrt{x+\sqrt{x}}+2}{\sqrt{x+\sqrt{x}}+2}$

$= \frac{(x+\sqrt{x}-8)(\sqrt[3]{x^2}+1+\sqrt[3]{x})}{(x-1)\times\sqrt{x+\sqrt{x}+2}} = \frac{(\sqrt{x}-1)(x+2)\times(\sqrt[3]{x^2}+1+\sqrt[3]{x})}{(x-1)\cdot\sqrt{x+\sqrt{x}+2}}$

$(x-1)$ $(\sqrt{x}+1)$

$\frac{21}{8}$ توجه

$= \frac{7}{2} \times \frac{3}{4}$

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8) $\lim_{x\to \pi} \frac{1+\cos^3 x}{\sin^2 x} = \frac{(1+\cos x)(1+\cos^2 x - \cos x)}{(1-\cos x)(1+\cos x)} = \frac{1+1+1}{2} = \frac{3}{2}$

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9) $\lim_{x\to \frac{\pi}{4}} \frac{1-\tan x}{\sin x - \cos x} = \frac{\cos x - \sin x}{\cos x} = \frac{-1}{\cos x} = \frac{-1}{\cos(\frac{\pi}{4})} = \frac{-2}{\sqrt{2}}$

$\to \frac{-2\times\sqrt{2}}{2} = -\sqrt{2}$

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10) $\lim_{x\to \frac{3}{4}\pi} \frac{\tan^2 x - 1}{\cos 2x} = \frac{\tan^2 \frac{3}{4}\pi - 1}{\cos 2\times\frac{3}{4}\pi} = \frac{(\tan x - 1)(\tan x + 1)}{(\cos^2 x - \sin^2 x)}$

$\to \frac{(\sin x - \cos x)(\sin x + \cos x)}{(\cos x + \sin x)(\cos x - \sin x)} = \frac{-1}{\cos^2(\frac{3}{4}\pi)} = \frac{-1}{(-\frac{\sqrt{2}}{2})^2} = \frac{-1}{\frac{1}{2}}$

$= -2$