۱) $f(x)=\begin{cases}\cot\frac{\pi x}{4} & x\le 1\\ \sqrt{x^2+1} & x>1\end{cases}$ $foF(\frac{\pi}{2})=?$ $F(F(\sqrt{2}))=F(\sqrt{2})=2$ جواب

$\frac{\pi}{2}\le 1 \to \cot\frac{\pi\times 2}{4}=\cot\frac{\pi}{2}=\sqrt{2}$

$\sqrt{2}>1 \to \sqrt{(\sqrt{2})^2+1}=\sqrt{4}=2$

۲) الف) اگر $f(\frac{1}{2m})=\sqrt{\frac{m+1}{m^2}}$ ، $g(x)=2\cos^2 x$ مقدار $f(g(\frac{\pi}{3}))$ را بیابید و دهید.

$g(\frac{\pi}{3})\to 2\cos^2\frac{\pi}{3}=2\times\frac{1}{4}=\frac{1}{2}\to f(\frac{1}{2})\to x=2=\sqrt{\frac{4+4}{4}}=\sqrt{\frac{8}{4}}=\frac{\sqrt{4}}{2}$ جواب

۳) $f(x)=\sin x$ $g(x)=x\sqrt{1-x^2}$ $gof(\frac{\pi}{3})=?$ $g(f(\frac{\pi}{3}))=g(\frac{\sqrt{3}}{2})=\frac{1}{2}$ جواب

$f(\frac{\pi}{3})=\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$ $g(\frac{\sqrt{3}}{2})=\frac{\sqrt{3}}{2}\sqrt{1-\frac{3}{4}}=\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}=\frac{1}{2}$

۴) $f(x)=\{(3,5)(4,5)(8,12)(10,12)\}$ $g(x)=\{(3,4)(2,3)(4,8)(8,10)\}$

الف) $fog(x)=f(g(x))$ $3\to 4\to 5$, $2\to 3\to 5$, $4\to 8\to 12$, $8\to 10\to 12$ $=\{(3,5),(2,5)(4,12)(8,12)\}$

ب) $gof(x)=g(f(x))$ $3\to 5\to\alpha$, $4\to 5\to\alpha$, $8\to 12\to\alpha$, $10\to 12\to\alpha$

ج) $fof(x)=f(f(x))$ $3\to 5\to\alpha$, $4\to 5\to\alpha$, $8\to 12\to\alpha$, $10\to 12\to\alpha$

د) $gog(x)=g(g(x))$ $2\to 4\to 8$, $4\to 8\to 10$, $8\to 10\to\alpha$, $10\to\alpha$ $\{(3,8)(2,4)(4,10)\}$

۵) $F=\{(2,1)(3,2)(3,5)(1,4)\}$ $g=\{(1,2)(3,1)(\alpha,3)(b,1)\}$ $(3,2)\in F(g(x))$ $(a,b)=?$

$F(g(x))=\{(1,1)(3,4)(\alpha,2)(b,4)\}$

$1\to 2\to 1$, $3\to 1\to 4$, $9\to 3\to 2\to a=3$, $b\to 1\to 4$

$g(F(x))=$

$2\to 1\to 2$, $3\to 2\to$, $3\to 5\to 1$, $1\to 4\to$

$(3,1)\in g(F(x))$

چون $(3,1)$ داده‌ی سوم مجموعه‌ی g برابری برابر با $b=3$ می‌شود

$(a,b)=(3,3)$

۶- $f, g \to$ تابع خطی $\quad f(f(x)) = 4x+3 \quad g(2x+3) = 2x-2 \quad gof(-1) = ?$

$g(f(-1))$

$f(x) = ax+b \quad g(x) = cx+d$

$a(ax+b)+b = 4x+3 \quad d(2x+3)+c = 2x-2 \quad g(-1) =$

$a^2x+ab+b = 4x+3 \quad 2cx+3d+c = 2x-2 \quad \frac{3}{2}(-1) - \frac{13}{2} = -\frac{16}{2} = -8$ جواب

$a^2 = 4 \quad ab+b = 3 \quad 2c = 2 \quad c = \frac{2}{3}$

$a = \pm 2 \quad b(a+1) = 3 \quad \frac{9}{2} + d = -3 \Rightarrow -\frac{6}{2}$

$a = 2 \quad b = 1 \quad d = -\frac{13}{2}$

$2(-1)+1 = -1 \xleftarrow{x=-1} f(x) = 2x+1$

$2-4 = -1 \xleftarrow{x=-1} f(x) = -2x-4 \leftarrow a=-2 \quad b=-4$

$g(x) = \frac{3}{2}x - \frac{13}{2}$

۷-

$f(x) = \sqrt{x+|x|}$ $\quad g(x) = \frac{1}{x^2-4x}$ $\quad D_{gof} = ?$ $\quad (0,+\infty) - \{4\}$

$x+|x| \geq 0 \quad |x| \geq -x$ $\quad x(x-4) \neq 0 \quad x \neq 0, 4$

$g(f(x))$

$\{x \in D_f, f(x) \in D_g\} \to x \in \mathbb{R}, f(x) \in \mathbb{R} - \{0, 4\}$

$\sqrt{x+|x|} \neq 0$ ① $\quad -x \neq |x| \quad x > 0$ $\quad \sqrt{x+|x|} \neq 4$ ② $\quad x+|x| \neq 16 \to x \neq 8$

۸- $f(x) = \sqrt{1-x^2} \quad g(x) = \sqrt{x}$ $\quad D_{(f+g)of} = ? \to fof + gof = f(f(x)) + g(f(x))$

$\sqrt{1-(\sqrt{1-x^2})^2} + \sqrt{\sqrt{1-x^2}} \to 1-x^2 \geq 0 \quad 1 \geq x^2$ ① $\quad -1 \leq x \leq 1$ $\quad [-1,1]$ جواب

$1-(\sqrt{1-x^2})^2 \geq 0 \quad 1 \geq (\sqrt{1-x^2})^2 \quad 1 \geq \sqrt{1-x^2} \geq -1 \to 1 \geq 1-x^2 \quad x^2 \geq 0$ ②

بدون

۹- الف) $f\left(\frac{3x+1}{x-2}\right) = 4x+5 \quad \frac{3x+1}{x-2} = t \quad x = -\frac{-2t-1}{t-3} = \frac{2t+1}{t-3} \quad f(x) = ?$

$f(t) = 4\left(\frac{2t+1}{t-3}\right) + 5 = \frac{8t+4+5t-15}{t-3} = \frac{13t-11}{t-3} \to f(x) = \frac{13x-11}{x-3}$

ب) $f(x+\frac{1}{x}) = x^3 + \frac{1}{x^3} \to \left(x+\frac{1}{x}\right)^3 - 3\left(x+\frac{1}{x}\right) \Rightarrow f(x) = x^3 - 3x$

۱۰- $g(x) = a(x-2\sqrt{2})(x-1)$ $\quad g(f(x)) =$ محور طول‌ها را قطع کرده یعنی $y=0$ شده

یعنی به ازای $f(x)$ جواب‌هایی که از g حاصل می‌شود

$g(f(x)) = 0 \to f(x) = 1 = x\sqrt{x} = 1 \xrightarrow{\text{توان ۲}} x^2 \times x = 1 \quad x^3 = 1 \quad x = 1$

$f(x) = 2\sqrt{2} \quad x\sqrt{x} = 2\sqrt{2} \quad x = 2$

اختلاف طول‌ها $2 - 1 = 1$

$f_{\circ g}(\sqrt{2})$

$f(g(\sqrt{2}))$

$f(-\sqrt{2}-2)$

$g(x) = \frac{x}{1-x}$ $\quad f(x) = [x]$

$g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}(1+\sqrt{2})} = -(\sqrt{2})(1+\sqrt{2}) = -\sqrt{2}-2$

$\frac{1-2}{-1}$

$\hookrightarrow f(-\sqrt{2}-2) = [-2-\sqrt{2}] = -3$

$\underbrace{-3.4}_{-4}$

$f_{\circ g}(\sqrt{2}) = -3$

ب - ۲