

۲. آنتین

الف) $6x^2 - 5x - 6 \rightarrow x^2 - 5x - 36 = (x-9)(x+4) \rightarrow (x-\frac{9}{6})(x+\frac{4}{6}) = 0$
 $x = \frac{9}{6} = \frac{3}{2}$ $x = -\frac{4}{6} = -\frac{2}{3}$

ب) $3x^2 - 11x - 2 \rightarrow x^2 - 11x - 9 = (x-9)(x+1) \rightarrow (x-\frac{9}{3})(x+\frac{1}{3}) = 0$
 $x = \frac{9}{3} = 3$ $x = -\frac{1}{3}$

ج) $4x^2 - 4x - 3 \rightarrow x^2 - 4x - 12 = (x-6)(x+2) \rightarrow (x-\frac{6}{4})(x+\frac{2}{4}) = 0$
 $x = \frac{6}{4} = \frac{3}{2}$ $x = -\frac{2}{4} = -\frac{1}{2}$

د) $2x^2 - 5x - 12 \rightarrow x^2 - 5x - 24 = (x-1)(x+3) \rightarrow (x-\frac{1}{2})(x+\frac{3}{2}) = 0$
 $x = \frac{1}{2}$ $x = -\frac{3}{2}$

الف) $x^2 - 5x - 14 = (x-7)(x+2) = 0 \Rightarrow x-7=0 \quad x+2=0$
 $x=7$ $x=-2$

ب) $2x^2 + 5x + 2 \Rightarrow x^2 + 5x + 6 = 0 = (x+2)(x+3) \rightarrow (x+\frac{2}{2})(x+\frac{3}{2}) = 0$
 $x=-1$ $x=-\frac{3}{2}$

ج) $x^2 + 2x + 3 \quad \Delta = b^2 - 4ac = 4 - 12 = -8 < 0 \Rightarrow$ ریشه ندارد

د) $3x^2 + 3x - 2 = 0 \quad \Delta = b^2 - 4ac = 9 + 24 = 33$
 $x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-3 \pm \sqrt{33}}{6}$

$A \times B = \{(1,2), (1,3), (2,3), (3,2), (3,3)\}$

$B \times B = \{(2,3), (3,2), (3,3)\}$

$A \cap B = \{(1,2), (1,3)\}$

الف) $f(x) = \{(3, m^2), (2, 1), (-3, m), (-2, m), (3, m+2), (m, 4)\}$
 $m^2 = m+2 \Rightarrow m^2 - m - 2 = 0 \quad (m-2)(m+1) = 0$
 $m=2$ $m=-1$

ب) $g(x) = \{(3, m^2), (2, 1), (m, 4), (3, m+2), (-2, m), (-1, 3)\}$
 $m^2 = m+2 \Rightarrow m^2 - m - 2 = 0 \quad (m-2)(m+1) = 0$
 $m=2$ $m=-1$

$f(x) = \{(1, a^2), (1, 3a-2), (a^2, 3a+2)\}$

$a^2 = 3a-2 \Rightarrow a^2 - 3a + 2 = 0$

$(a-2)(a-1) = 0$

$(1, 4), (4, 1) \leftarrow a=2$

غیر تابع نیست $(1, 1), (1, 5) \leftarrow a=1$

$$f(x) = \begin{cases} x^2 - 1 & x \leq 1 \\ ax + b & 1 \leq x \leq 2 \\ x^3 & x \geq 2 \end{cases}$$

$$x=2 \Rightarrow y = (2)^3 = (2)a + b \\ 2a + b = 8$$

$$x=1 \Rightarrow y = (1)^2 - 1 = (1)a + b \\ \begin{cases} 2a + b = 8 \\ -a + b = 0 \end{cases} \Rightarrow \begin{cases} a = 1 \\ b = -1 \end{cases} \\ a \times b = 1 \times (-1) = -1$$

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$$f(x) = \begin{cases} 2x - 3 & x \geq k \\ 8 - 3x & x \leq k \end{cases}$$

$$x=k \Rightarrow y = 2(k) - 3 = 8 - 3(k)$$

$$2k - 3 = 8 - 3k$$

$$5k = 11$$

$$k = \frac{11}{5}$$

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$$\text{الف) } y^3 - 4y = x^2 - 4x + 3 \quad x=1 \Rightarrow y^3 - 4y = 0 \quad y^3 = 4y \quad y^2 = 4$$

$$y = \pm \sqrt{2}$$

$$\text{ب) } |x| + |y| = 3 \quad x=0 \Rightarrow |y| = 3$$

$$y = \pm 3$$

$$\text{ج) } |y| = \sqrt{1 - \cos x}$$

$$x = \pi \Rightarrow |y| = \sqrt{2}$$

$$y = \pm \sqrt{2}$$

$$y^2 + x^2 + 2x + 1 = 0$$

$$y^2 + (x+1)^2 = 0$$

$$x = -1, y = 0$$

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$$\text{الف) } \sqrt[3]{y} + x^2 = 2x - 2 \quad \sqrt[3]{y_1} = -x^2 + 2x - 2 \quad \sqrt[3]{y_2} = \sqrt[3]{y_1} \Rightarrow y_1 = y_2$$

$$\sqrt[3]{y_2} = -x^2 + 2x - 2$$

$$y_1^3 + 2y_2 = y_1^3 + 2y_1 \quad y_1^3 - y_2^3 = 2y_2 - 2y_1$$

$$(y_1 - y_2)(y_1^2 + y_1y_2 + y_2^2) = -2(y_1 - y_2)$$

$$(y_1 - y_2)(y_1^2 + y_1y_2 + y_2^2 + 2) = 0$$

$$(y_1 - y_2)(y_1^2 + y_1y_2 + y_2^2 + 2) = 0$$

$$y_1 - y_2 = 0 \Rightarrow y_1 = y_2$$

$$y_1^2 + y_1y_2 + y_2^2 + 2 = 0$$

$$1 = y_2^2 - 4(y_2^2 + 2) = -3y_2^2 - 8 \Rightarrow 0 < 0$$

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$$\text{الف) } x \sin y = y \sin x \quad x=0 \rightarrow 0 \times \sin y = 0 \times y \quad y = 1, \pi, 2\pi, \dots$$

$$y = 1, \pi, 2\pi, \dots$$

$$\text{ب) } \sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = 4$$

$$\frac{x^2 + y^2}{xy} = 4$$

$$x^2 + y^2 = 4xy$$

$$x^2 - 2xy + y^2 = 0$$

$$(x - y)^2 = 0 \Rightarrow x = y$$

$$\frac{x}{y} + \frac{y}{x} + 2\sqrt{\frac{x}{y} \cdot \frac{y}{x}} = 4 \Rightarrow \frac{x}{y} + \frac{y}{x} = 2$$

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سؤال 10: $x \neq y$ یا نه؟

(۱۰-ب) $\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = 2$

این عبارت تنها زمانی به قدر است که $\sqrt{\frac{x}{y}} = 1$ باشد پس

$\sqrt{\frac{x}{y}} = 1 \leadsto \frac{x}{y} = 1 \leadsto x = y$ و $(x, y \neq 0)$

