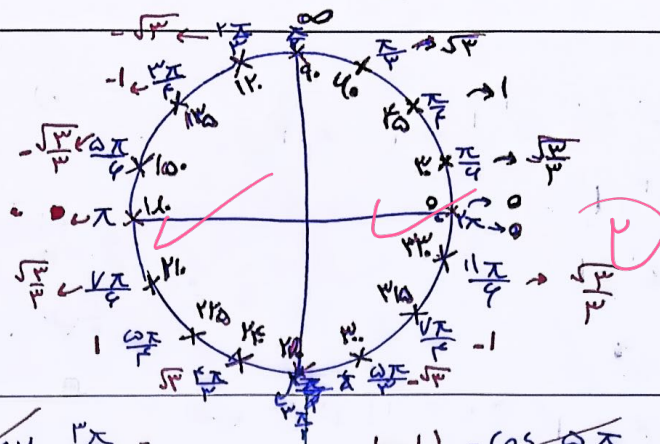


۱۹ آفرین



1

الف) $\sin \frac{\pi}{4} + \cos \pi + 2 \tan \frac{3\pi}{4} =$
 $\frac{\pi}{4} = 45^\circ \quad \downarrow \frac{1}{\sqrt{2}}$
 $\pi = 180^\circ$
 $\frac{3\pi}{4} = 135^\circ$
 $\frac{1}{\sqrt{2}} + (-1) + 2(-\frac{1}{\sqrt{2}}) = -\frac{1}{\sqrt{2}}$

ب) $-\cos \frac{5\pi}{4} + \sin \frac{\pi}{4} - 3 \sin \frac{\pi}{3}$
 $\frac{5\pi}{4} = 225^\circ \quad +\frac{\sqrt{2}}{2} \quad \frac{\sqrt{3}}{2}$
 $\frac{\pi}{4} = 45^\circ$
 $\frac{\pi}{3} = 60^\circ$
 $-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 3 \cdot \frac{\sqrt{3}}{2} = -\frac{3\sqrt{3}}{2}$

2

ج) $\cot \frac{\pi}{4} + 3 \cot \frac{\pi}{3} - 2 \tan \frac{5\pi}{4}$
 $\frac{\pi}{4} = 45^\circ \quad -\frac{\sqrt{3}}{3}$
 $\frac{\pi}{3} = 60^\circ \quad +3(\frac{\sqrt{3}}{3})$
 $\frac{5\pi}{4} = 225^\circ \quad -2(-\frac{\sqrt{3}}{3}) = \frac{2\sqrt{3}}{3}$

د) $\cos(\sin \frac{\pi}{4}) + \sin(\pi \cos \frac{\pi}{4})$
 $\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$
 $\cos(\frac{\sqrt{2}}{2}) + \sin(\pi \cdot \frac{\sqrt{2}}{2})$
 $\cos(0) = 1$
 $\sin(\frac{\pi}{2}) = 1$
 $1 + 1 = 2$

3

$x=a: a^2 + 4a = 2a + 2$
 $a^2 + a - 2 = 0$
 $(a+2)(a-1)$
 $a \neq -2 \Rightarrow a=1$

$f(x) = \begin{cases} x^2 + 2x & x > a \\ 2x + a + 2 & x \leq a \end{cases}$

حال در $x=1$ از معادله داریم
 $\Rightarrow x^2 + 4x: x=1$
 $f(1)=5$

4

$f(x) = \begin{cases} 2x^2 - b & |x+1| \geq 2 \\ a + 2x & -3 \leq x \leq -1 \end{cases}$
 $|x+1| \geq 2$
 $x+1 \geq 2 \Rightarrow x \geq 1$
 $x+1 \leq -2 \Rightarrow x \leq -3$

$2x^2 - b = a + 2x$
 $11 - b = a - 4$
 $2f = a + b$

5

الف) $\frac{1}{x^3 - 4x^2}$
 $x^3 - 4x^2 > 0$
 $x^2(x-4) > 0$
 $x \neq 0$
 $x > 4$

ب) $\frac{1}{2x^3 + 4x^2 + 2x + 4}$
 $2x^3 + 4x^2 + 2x + 4 \neq 0$
 $x^3 + 2x^2 + x + 2 \neq 0$
 $(x^3 + 2x^2) + (x + 2)$
 $x^2(x+2) + (x+2)$
 $(x+2)(x^2 + 1) \neq 0$
 $x \neq -2$
 $x^2 + 1 \neq 0$

الف) $y = \sqrt{x^2 - 1} + \sqrt{x - 2}$

$x^2 - 1 \geq 0$
 $x^2 \geq 1$
 $x \geq 1$ یا $x \leq -1$

$x - 2 \geq 0$
 $x \geq 2$

پس $x \geq 2$

ب) $y = \frac{\sqrt{4 - 3x}}{\sqrt{9x - 3}}$

$4 - 3x \geq 0$
 $x \leq \frac{4}{3}$

$9x - 3 \geq 0$
 $x \geq \frac{1}{3}$

پس $\frac{1}{3} \leq x \leq \frac{4}{3}$

6

الف) $4x - a \geq 0$

$4x \geq a$
 $x \geq \frac{a}{4}$

ب) $b - 3x \geq 0$

$b \geq 3x$
 $x \leq \frac{b}{3}$

پس $\frac{a}{4} \leq x \leq \frac{b}{3}$

7

الف) $\frac{x^2 - 2x - 4}{x^3 - 13x^2 + 34x}$

$x^2 - 2x - 4 \geq 0$
 $x = \frac{2 \pm \sqrt{4 + 16}}{2} = 1 \pm \sqrt{5}$

ب) $\frac{x^3 - 2x^2 - 5x + 4}{x^3 + 2x^2 - 20x - 4}$

$x^3 - 2x^2 - 5x + 4 = (x - 4)(x + 1)(x - 1)$
 $x^3 + 2x^2 - 20x - 4 = (x + 4)(x + 1)(x - 2)$

پس $x \in (-\infty, -4) \cup (-1, 1) \cup (2, +\infty)$

8

الف) $\sqrt{2[x] - 4}$

$2[x] - 4 \geq 0$
 $[x] \geq 2$
 $x \geq 2$

ب) $\sqrt{2[x] - 5}$

$2[x] - 5 \geq 0$
 $[x] \geq \frac{5}{2}$
 $x \geq \frac{5}{2}$

9

الف) $\frac{x+1}{|2x-3|} + \sqrt{\frac{4-x}{x^2+2x+4}}$

$\frac{x+1}{|2x-3|} \geq 0$
 $x+1 \geq 0$ یا $x+1 \leq 0$ و $2x-3 < 0$ یا $2x-3 > 0$

$\frac{4-x}{x^2+2x+4} \geq 0$
 $4-x \geq 0$
 $x \leq 4$

پس $x \in [-1, 4]$

ب) $\sqrt{[2x] - 4} + \sqrt{2 - [x]}$

$[2x] - 4 \geq 0$
 $[2x] \geq 4$
 $x \geq 2$

$2 - [x] \geq 0$
 $[x] \leq 2$
 $x \leq 2$

پس $x = 2$

10

الف- ۸)

$$y = \sqrt{\frac{n^2 - n - 4}{n^2 - 13n^2 + 34}} \leadsto \frac{n^2 - n - 4}{n^2 - 13n^2 + 34} \geq 0 \leadsto \frac{(n-3)(n+2)}{(n^2-9)(n^2-4)} \geq 0$$

$$\frac{(n-3)(n+2)}{(n-3)(n+3)(n-2)(n+2)} \geq 0 \leadsto \begin{array}{ccccccc} & -3 & -2 & 2 & 3 & & \\ & * & * & & * & & \\ + & \frac{0}{+} & - & \frac{0}{-} & - & \frac{0}{+} & + \end{array}$$

$$D_f = (-\infty, -3) \cup (2, 3) \cup (3, +\infty)$$

ب- ۸) $y = \sqrt{\frac{n^3 - 2n^2 - 5n + 4}{n^3 + 2n^2 - 5n - 4}} \leadsto \frac{n^3 - 2n^2 - 5n + 4}{n^3 + 2n^2 - 5n - 4} \geq 0 \leadsto$

مخواب منفی پذیر (n-1)
مخواب دهن زوج و فرد برابر منفی پذیر (n+1)

$$\frac{(n-1)(n^2 - n - 4)}{(n+1)(n^2 + n - 4)} \geq 0 \leadsto \frac{(n-1)(n-3)(n+2)}{(n+1)(n+3)(n-2)} \geq 0 \leadsto \begin{array}{ccccccc} -3 & -2 & -1 & 1 & 2 & 3 & \\ + & \frac{0}{+} & - & \frac{0}{+} & - & \frac{0}{+} & + \end{array}$$

$$D_f = (-\infty, -3) \cup [-2, -1) \cup [1, 2) \cup [3, +\infty)$$