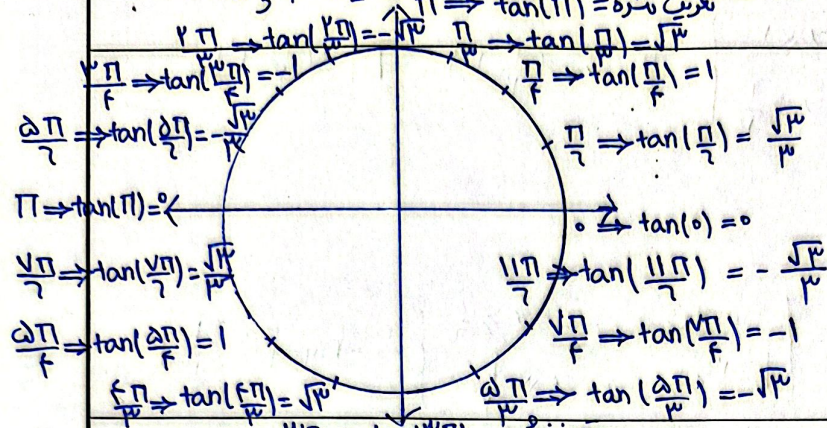


نام و نام خانوادگی کلاس شماره پاسنامه تشریحی تکلیف شماره ۲ ... بازدهی ...



۱

الف) $\sin \frac{\pi}{4} + \cos \pi + 2 \tan \frac{\pi}{4} = \sin \frac{\pi}{4} + \cos \pi + 2 \tan \frac{\pi}{4} = \frac{1}{\sqrt{2}} - 1 + 2(-1) = -\frac{\sqrt{2}}{2}$

ب) $-\cos \frac{\pi}{4} + \sin \frac{\pi}{4} - 3 \sin \frac{\pi}{4} = -\cos \frac{\pi}{4} + \sin \frac{\pi}{4} - 3 \sin \frac{\pi}{4} = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 3 \left(\frac{\sqrt{2}}{2} \right) = -\frac{5\sqrt{2}}{2}$

ج) $\cot \frac{\pi}{4} + 2 \cot \frac{\pi}{4} - 4 \tan \frac{\pi}{4} = \cot \frac{\pi}{4} + 2 \cot \frac{\pi}{4} - 4 \tan \frac{\pi}{4} = 1 + 2(1) - 4(1) = -1$

د) $\cos(\sin \pi) + \sin(\pi \cos \frac{\pi}{4}) = \cos(0) + \sin(\frac{\pi}{2}) = \cos 0 + \sin 90 = 1 + 1 = 2$

۳

ف) $f(x) = \begin{cases} x^2 + 2x & x \geq a \\ 2x + a + 2 & x < a \end{cases}$ در $x=a$ باید هر دو ضابطه مقدار یکسانی بدهند:

$$a^2 + 2a = 2a + a + 2 \rightarrow a^2 + 2a = 3a + 2 \rightarrow a^2 - a - 2 = 0$$

$$\rightarrow (a+2)(a-1) = 0 \Rightarrow \begin{cases} a = 1 \\ a = -2 \end{cases}$$

۳ وقت چون a می باشد

ف) $f(x) = \begin{cases} x^2 + 2x & x \geq 1 \\ 2x + 3 & x < 1 \end{cases} \Rightarrow f(1) = 1 + 2 = 3$

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ف) $f(x) = \begin{cases} 2x^2 - b & |x+1| \geq 2 \\ a + 2x & -3 \leq x \leq -1 \end{cases}$ $|x+1| \geq 2 \rightarrow x+1 \geq 2 \text{ یا } x+1 \leq -2 \Rightarrow x \geq 1 \text{ یا } x \leq -3$

ف) $f(x) = \begin{cases} 2x^2 - b & x \geq 1 \text{ یا } x \leq -3 \\ a + 2x & -3 \leq x \leq -1 \end{cases}$ در $x = -3$ باید هر دو ضابطه مقدار یکسانی بدهند:

$$18 - b = a - 6 \Rightarrow a + b = 24$$

۵

الف) $y = \frac{1}{x^3 - 2x^2} \rightarrow x^3 - 2x^2 = 0 \rightarrow x^2(x-2) = 0 \Rightarrow x = 0 \text{ یا } x = 2 \Rightarrow D_f = \mathbb{R} - \{0, 2\}$

ب) $y = \frac{1}{2x^3 + 4x^2 + 2x + 2} \rightarrow 2x^3 + 4x^2 + 2x + 2 = 0 \rightarrow 2x^2(x+2) + 2(x+1) = 0 \rightarrow (x+2)(2x^2 + 2) = 0$

چون $2x^2 + 2 > 0$ همواره مثبت است، تنها $x = -2$ است که $D_f = \mathbb{R} - \{-2\}$

الف) $y = \sqrt{x^2-1} + \sqrt{x-2} \rightarrow \begin{cases} x^2-1 \geq 0 \rightarrow x \leq -1 \vee x \geq 1 \\ x-2 \geq 0 \rightarrow x \geq 2 \end{cases} \rightarrow (1, \infty, -1) \cup [1, +\infty) \cap [2, +\infty) = [2, +\infty) \Rightarrow Df = [2, +\infty)$	
ب) $y = \frac{\sqrt{7-3x}}{\sqrt{2x-3}} \rightarrow \begin{cases} 7-3x \geq 0 \rightarrow x \leq \frac{7}{3} \\ 2x-3 > 0 \rightarrow x > \frac{3}{2} \end{cases} \xrightarrow{\cap} x \in (\frac{3}{2}, \frac{7}{3}] \rightarrow Df = (\frac{3}{2}, \frac{7}{3}]$	٦
$y = \sqrt{7x-a} + \sqrt{b-3x} \quad Dy =]1[\rightarrow \begin{cases} 7x-a \geq 0 \rightarrow x \geq \frac{a}{7} \\ b-3x \geq 0 \rightarrow x \leq \frac{b}{3} \end{cases} \xrightarrow{\cap} Dy = [\frac{a}{7}, \frac{b}{3}]$ $\frac{a}{7} = 2 \Rightarrow a = 14$ $\frac{b}{3} = 2 \Rightarrow b = 6 \Rightarrow \frac{b}{a} = \frac{6}{14} = \frac{3}{7}$	٧
الف) $y = \sqrt{\frac{x^2-x-7}{x^4-13x^2+37}} \rightarrow \frac{x^2-x-7}{x^4-13x^2+37} \geq 0 \rightarrow \frac{(x-3)(x+2)}{(x^2-9)(x^2-4)} \geq 0 \rightarrow \frac{-3-2}{+ - - + +}$ $\Rightarrow Dy = (-\infty, -3) \cup (2, 3) \cup (3, +\infty)$ ب) $y = \sqrt{\frac{x^3-2x^2-5x+7}{x^3+2x^2-5x-7}} \rightarrow \frac{x^3-2x^2-5x+7}{x^3+2x^2-5x-7} \geq 0 \rightarrow \frac{(x-1)(x^2-x-7)}{(x+1)(x^2+x-7)} \geq 0$ $\frac{(x-1)(x-3)(x+2)}{(x+1)(x+3)(x-2)} \geq 0 \rightarrow \frac{-3-2-1+2}{+ - - + - - +} \Rightarrow Dy = (-\infty, -3) \cup [3, +\infty) \cup [2, 1) \cup (1, 2)$	٨
الف) $y = \sqrt{2[x]-4} \rightarrow 2[x] \geq 4 \rightarrow [x] \geq 2 \Rightarrow x \geq 2 \rightarrow Dy = [2, +\infty)$ ب) $y = \sqrt{2[x]-5} \rightarrow 2[x]-5 \geq 0 \rightarrow 2[x] \geq 5 \rightarrow [x] \geq \frac{5}{2} \Rightarrow x \geq 3 \rightarrow Dy = [3, +\infty)$	٩
الف) $y = \sqrt{\frac{x+1}{ x-3 }} + \sqrt{\frac{7-x}{x^2+2x+4}} \rightarrow \begin{cases} \frac{x+1}{ x-3 } \geq 0 \rightarrow \frac{-1}{- + +} \rightarrow x \in [-1, 3) \cup (3, +\infty) \\ \frac{7-x}{x^2+2x+4} \geq 0 \rightarrow \frac{7-x}{(x+1)^2+3} \geq 0 \rightarrow 7-x \geq 0 \Rightarrow x \leq 7 \end{cases} \xrightarrow{\cap} Dy = [-1, 3) \cup (3, 7]$ ب) $y = \sqrt{[x]-4} + \sqrt{2-[x]} \rightarrow \begin{cases} [x]-4 \geq 0 \rightarrow [x] \geq 4 \rightarrow x \geq 4 \\ 2-[x] \geq 0 \rightarrow [x] \leq 2 \rightarrow x < 3 \end{cases} \xrightarrow{\cap} Dy = \emptyset$	١٠