

حل المسألة

أ) $x^4 + 4x^3 - 12x^2 = (x-1)(4x^3 + 8x^2 - 8x - 4) \rightarrow$ $\text{نحل: } \frac{1}{4} > -4 \Rightarrow D_A = \mathbb{R} - \{\frac{1}{4}, -4\}$ (1)

ب) $x^4 + 9x^3 + 10x^2 = (x+1)(x^3 + 8x^2 + 10x + 10) \rightarrow$ $\text{نحل: } -4 > -\frac{1}{4} \Rightarrow D_A = \mathbb{R} - \{-4, -\frac{1}{4}\}$

ج) $x^4 - 2x^3 + 2x - 1 = (x-1)(x^3 - x^2 + 2x - 1) \Rightarrow D_A = \mathbb{R} - \{1\}$ (2)

د) $\frac{x^4 - 2x^3 + 2x - 1}{x^3 - 2x^2 + 2x - 1} \geq 0 \Rightarrow \frac{-x}{x^3 - 2x^2 + 2x - 1} \geq 0 \Rightarrow D_A = \mathbb{R} - \{-4, 1\}$

$x^3 - 2|x-1| - 2x + 8 \neq 0 \rightarrow \begin{cases} x > 1 \rightarrow x^3 - 2x + 10 \neq 0 \\ x = 1 \rightarrow x^3 - 2x + 8 \neq 0 \\ x < 1 \rightarrow x^3 + 2x \neq 0 \end{cases} \Rightarrow D_A = \mathbb{R} - \{0, -4, 1\}$ (3)

هـ) $[x] \geq 1 \Rightarrow -x \geq 1 \Rightarrow D_A = (-\infty, -1]$ (4)

و) $[x]^2 - 2[x] - 3 \neq 0 \Rightarrow [x] \neq 3, [x] \neq -1 \Rightarrow \begin{cases} [x] \neq 3 \rightarrow x \geq 0, x < -1 \\ [x] \neq -1 \rightarrow x \geq 1, x < 0 \end{cases}$

$\Rightarrow D_A = (-\infty, -1) \cup [0, 3) \cup [1, +\infty)$

ز) $[x - \frac{1}{4}] + [x + \frac{3}{4}] \geq 0 \rightarrow [x - \frac{1}{4}] + 1 + [x + \frac{3}{4}] - 1 \geq 0 \rightarrow 2[x + \frac{3}{4}] - 1 \geq 0 \rightarrow [x + \frac{3}{4}] \geq \frac{1}{2}$ (5)

$x \geq \frac{1}{4} \Rightarrow D_A = [\frac{1}{4}, +\infty)$

ح) $[x - \frac{1}{4}] + [\frac{1}{4} - x] \geq 0 \rightarrow [t] - [t] \geq 0 \rightarrow t \in \mathbb{Z} \rightarrow x \in \mathbb{Z} + \frac{1}{4} \rightarrow D_A = \{x | x = k + \frac{1}{4}, k \in \mathbb{Z}\}$ (6)

ط) $\sin x \neq 0, \cos x \neq 0, \tan x \neq -1 \Rightarrow D_A = \mathbb{R} - \{\frac{k\pi}{2}, k\pi - \frac{\pi}{4}\}$ (7)

ي) $1 - \frac{\sin^2 x}{\sin^2 x} \geq 0 \rightarrow \sin^2 x \leq \frac{1}{4} \rightarrow -\frac{1}{2} \leq \sin x \leq \frac{1}{2} \Rightarrow D_A = [k\pi - \frac{\pi}{6}, k\pi + \frac{\pi}{6}]$ (8)

ق) $2\sin x - 1 \geq 0 \rightarrow \sin x \geq \frac{1}{2} \Rightarrow D_A = [2k\pi + \frac{\pi}{6}, 2k\pi + \frac{5\pi}{6}]$

ج) $1 - 2\cos x \geq 0 \rightarrow \cos x \leq \frac{1}{2} \Rightarrow D_A = [2k\pi + \frac{2\pi}{3}, 2k\pi - \frac{2\pi}{3}]$

د) $-1 \leq x^2 + 4x + 1 \leq 1 \rightarrow 0 \leq x^2 + 4x + 1 \leq 1 \rightarrow \frac{-4 \pm \sqrt{16 - 4}}{2} \leq x \leq \frac{-4 \pm \sqrt{16 - 4}}{2} \Rightarrow D_A = [-4 - 1, -4 + 1] \cup [-1, 1]$ (9)

هـ) $-1 \leq \sqrt{8 - 2x} \leq 1 \rightarrow 0 \leq 8 - 2x \leq 1 \rightarrow x \leq \frac{7}{2} \Rightarrow D_A = [2, \frac{7}{2}]$

الف) $1 - \log_{\frac{1}{2}} x > 0 \Rightarrow \log_{\frac{1}{2}} x < 1 \Rightarrow x < \frac{1}{2} \Rightarrow D_A = (0, \frac{1}{2})$
 $\log_{\frac{1}{2}} x \rightarrow x > 0$

ب) $x > 0$
 $1 - \log_{\frac{1}{2}} x > 0 \Rightarrow \log_{\frac{1}{2}} x < 1 \rightarrow x > \frac{1}{2} \Rightarrow D_A = (\frac{1}{2}, +\infty)$

الف) $2^{\epsilon x - x^2} - 1 \geq 0 \rightarrow 2^{\epsilon x - x^2} \geq 1 \Rightarrow \epsilon x - x^2 \geq 0 \rightarrow x^2 - \epsilon x + \epsilon \leq 0 \rightarrow \frac{1 - \sqrt{1 - 4\epsilon}}{2} \leq x \leq \frac{1 + \sqrt{1 - 4\epsilon}}{2} \rightarrow D_A = [\frac{1 - \sqrt{1 - 4\epsilon}}{2}, \frac{1 + \sqrt{1 - 4\epsilon}}{2}]$ (i.o)

ب) $\frac{x^2 + d}{x^2 + \epsilon} \in W \rightarrow x \in -\frac{\epsilon W - d}{x^2 - W} \rightarrow D_A = \{x \mid x = \frac{\epsilon k + d}{x^2 - W}, k \in W\}$