

$$\begin{aligned} \text{الف) } x+3 & \quad \frac{x^3 + 3x^2 - 12x + 3}{x^3 + 3x^2 - 12x + 3} \rightarrow \text{جمع ضرب} \\ & \quad \frac{-12x + 3}{-12x + 3} = 1 \\ & \quad R = \emptyset \end{aligned}$$

الف) $\frac{x+3}{x^3-2x^2+2x-1}$ ← جمع ضرب مقلوب

$\frac{x^3-2x^2+2x-1}{x^2-x+1} \begin{array}{r} x-1 \\ -x^2+x \\ +x^2-x \\ \hline x-1 \\ =x-1 \end{array}$

$\frac{x+3}{(x-1)(x^2-x+1)}$

$\Delta < 0$ ← ریشه ندارد
مقدار مثبت

$R = \{1\}$

$\frac{x+3}{x^3-2x^2+2x-1} \geq 0$

$\frac{x+3}{(x-1)(x^2-x+1)} \geq 0$ ← جمع ضرب مقلوب

$\frac{-3}{+6-\frac{1}{6}+}$

$((-\infty, -3] \cup (1, +\infty))$

$$y = \frac{r}{x^2 - 0|x-1| - 4x + 6}$$

$$y = \frac{r}{x^2 - 4x + 6} \quad x \geq 0 \rightarrow \frac{r}{(x-0)(x-4)} \quad \begin{matrix} x \neq 0 \\ x \neq 4 \end{matrix}$$

$$y = \frac{r}{x^2 + 4x} \quad x \leq 0$$

$$\frac{r}{x(x+4)} \quad \begin{matrix} x \neq 0 \\ x \neq -4 \end{matrix}$$

$$R = \{-r, 0, r, 6\}$$

$$\begin{aligned} \text{الف) } y &= \sqrt{[-x]-2} \quad [-x] \geq 2 && -x \geq 2 \quad (-\infty, -2] \\ & && x \leq -2 \\ \Rightarrow y &= \frac{0x^2+4}{[x]^2-2[x]-3} = \frac{0x^2+4}{([x]-3)([x]+1)} && [x] \neq 3 \\ & && x \notin [3, 4) \\ & && [x] \neq -1 \\ & && x \notin [-1, 0) \end{aligned}$$

الف) $y = \sqrt{\left[x - \frac{1}{x}\right] + \left[x + \frac{x}{x}\right]} \rightarrow \left[x - \frac{1}{x}\right] + \left[x + \frac{x}{x}\right] \geq 0$ $\left[x + \frac{x}{x}\right] - 1 + \left[x - \frac{1}{x}\right] \geq 0$
 $2\left[x + \frac{x}{x}\right] \geq 1$ $\left[x + \frac{x}{x}\right] \geq \frac{1}{2}$ $x + \frac{x}{x} \geq \frac{1}{2}$ $x \geq \frac{1}{x}$ $\boxed{\left[\frac{1}{x}, +\infty\right)}$

ب) $y = \sqrt{\left[x - \frac{1}{x}\right] + \left[\frac{1}{x} - x\right]}$ if $\begin{cases} x - \frac{1}{x} \in \mathbb{Z} & \left[x - \frac{1}{x}\right] + \left[\frac{1}{x} - x\right] = 0 & x - \frac{1}{x} \in \mathbb{Z} \\ x - \frac{1}{x} \notin \mathbb{Z} & \left[x - \frac{1}{x}\right] + \left[\frac{1}{x} - x\right] = -1 \end{cases}$
 $-(x - \frac{1}{x})$

$P_f = \left\{ x \mid x = k + \frac{1}{x}, k \in \mathbb{Z} \right\}$

$y = \frac{\cot x + 1}{\tan x + 1} = \frac{\frac{\cos x}{\sin x} + 1}{\frac{\sin x}{\cos x} + 1}$ $\sin x \neq 0 \quad x \neq k\pi$ $\cos x \neq 0 \quad x \neq k\pi + \frac{\pi}{2}$ $\tan x \neq -1 \quad x \neq k\pi - \frac{\pi}{4}$ $R = \left\{ \frac{k\pi}{4}, k\pi - \frac{\pi}{4} \right\}$	
$y = \sqrt{1 - \varepsilon \sin^2 x}$ $1 - \varepsilon \sin^2 x \geq 0$ $\varepsilon \sin^2 x \leq 1$ $\sin^2 x \leq \frac{1}{\varepsilon}$ $-\frac{1}{\sqrt{\varepsilon}} \leq \sin x \leq \frac{1}{\sqrt{\varepsilon}}$  $\left[k\pi - \frac{\pi}{\sqrt{\varepsilon}}, k\pi + \frac{\pi}{\sqrt{\varepsilon}} \right]$	8
$y = \sqrt{\gamma \sin x - 1}$ $\gamma \sin x - 1 \geq 0$ $\gamma \sin x \geq 1$ $\sin x \geq \frac{1}{\gamma}$  $\left[\gamma k\pi + \frac{\pi}{\gamma}, \gamma k\pi + \frac{\pi}{\gamma} \right]$	
$y = \sqrt{1 - \gamma \cos x}$ $1 - \gamma \cos x \geq 0$ $\gamma \cos x \leq 1$ $\cos x \leq \frac{1}{\gamma}$  $\left[\gamma k\pi + \frac{\pi}{\gamma}, \gamma k\pi + \frac{\pi}{\gamma} \right]$	9
$y = \arcsin(x^2 + \gamma x + 1)$ $-1 \leq x^2 + \gamma x + 1 \leq 1$ $x^2 + \gamma x \leq 0$ $x(x + \gamma) \leq 0$ $x^2 + \gamma x + \gamma \geq 0$ $(x + \gamma)(x + 1) \geq 0$ $\frac{-\gamma}{+\gamma - \gamma +} \quad \frac{-1}{+\gamma - \gamma +}$ $[-\gamma, -1] \cup [-1, 0]$ $\cos y = \sqrt{1 - \gamma x}$ $1 - \gamma x \geq 0$ $\gamma x \leq 1$ $x \leq \frac{1}{\gamma}$ $0 \leq \sqrt{1 - \gamma x} \leq 1$ $0 \leq 1 - \gamma x \leq 1$ $0 \leq \gamma x \leq 1$ $\frac{0}{\gamma} \leq x \leq \frac{1}{\gamma}$ $\left[\frac{0}{\gamma}, \frac{1}{\gamma} \right]$	10
$y = \log_{\gamma} (1 - \log_{\gamma} x)$ $x > 0$ $1 - \log_{\gamma} x > 0$ $\log_{\gamma} x < 1$ $x < \gamma$ $(0, \gamma)$ $y = \log_{\gamma} (1 - \log_{\gamma} \frac{1}{\gamma})$ $x > 0$ $1 - \log_{\gamma} \frac{1}{\gamma} > 0$ $\log_{\gamma} \frac{1}{\gamma} < 1$ $x > \frac{1}{\gamma}$ $\left(\frac{1}{\gamma}, +\infty \right)$	11
$y = \sqrt{\gamma^x - x^{\gamma}} - 1$ $\gamma^x - x^{\gamma} - 1 \geq 0$ $\gamma^x - x^{\gamma} \geq 1$ $\gamma^x - x^{\gamma} \geq \gamma$ $x^{\gamma} - \gamma x + \gamma \leq 0$ $(x - \gamma)(x - 1)$ $\frac{1}{+\gamma - \gamma +} \quad \frac{\gamma}{+\gamma - \gamma +}$ $[1, \gamma]$ $y = \left(\frac{\gamma x + \omega}{\gamma x + \varepsilon} \right)!$ $\frac{\gamma x + \omega}{\gamma x + \varepsilon} \in W$ $x \in -\frac{\varepsilon W - \omega}{\gamma W - \gamma}$ $\left\{ x \mid x = \frac{\omega - \varepsilon k}{\gamma k - \gamma}, k \in W \right\}$	12