

راه حل های د فرستمر را بخون و اشغال هات را تا به وقت امتحان بگیر

ایده های زیاد

۱۴ هنر و بهادرت مدال نه شود!

۱) $R = \{1, -\frac{1}{2}, -\frac{2}{3}\}$

$$y = \frac{n+3}{2n^3 + 3n^2 - n + 3} = \frac{(n+3)}{(n-1)(2n^2 + 5n + 3)}$$

تجزیه کنیم: $(n-1)(2n+3)(n+1)$

پس $y = \frac{1}{(n-1)(n+1)}$

۲) $R = \{-1, -3, -\frac{1}{2}\}$

$$y = \frac{n+3}{2n^3 + 9n^2 + 10n + 3} = \frac{n+3}{(n+1)(2n^2 + 7n + 3)}$$

تجزیه کنیم: $(n+1)(2n+3)(n+1)$

پس $y = \frac{1}{(n+1)^2}$

۳) $R = \{1\}$

$$y = \frac{n+3}{n^3 - 2n^2 + 2n - 1} = \frac{n+3}{(n^2 - n + 1)(n-1)}$$

۴) $y = \frac{n+3}{n^3 - 2n^2 + 2n - 1} \geq 0$

برای $n > 1$ داریم: $y > 0$

برای $n < 1$ داریم: $y < 0$

۵) $R = \{2, 0, 0\}$

$$y = \frac{2}{n^3 - 2n^2 + 2n - 1}$$

تجزیه کنیم: $(n-1)^2(n+1)$

ریشه ها چقدری پیدا شدند؟
راه حل تشریحی نوشته شود!

۶) $y = \sqrt{[-n] - 2} \Rightarrow [-n] - 2 \geq 0 \Rightarrow [-n] \geq 2 \Rightarrow (-\infty, -2]$

۷) $R = \{[3, 4] \text{ و } [1, 2]\}$

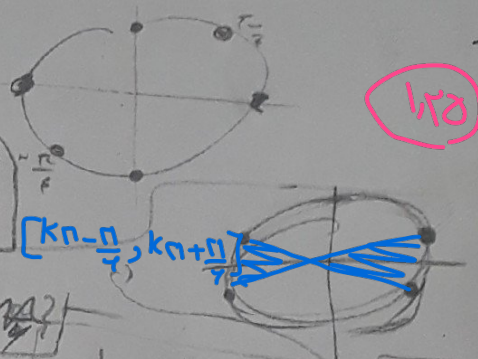
$$y = \frac{2n^2 + 4}{[n]^2 - 2[n] - 3} = \frac{(2n+4)}{([n]-3)([n]+1)}$$

۸) $\sqrt{[n - \frac{1}{2}] + [n + \frac{1}{2}]} \geq 0 \Rightarrow [n] - \frac{1}{2} + [n] + \frac{1}{2} \geq 0 \Rightarrow 2[n] \geq 0 \Rightarrow [n] \geq 0 \Rightarrow [0, +\infty)$

۹) $\sqrt{[n - \frac{1}{2}] + [\frac{1}{2} - n]} \geq 0 \Rightarrow [n] - \frac{1}{2} + [\frac{1}{2} - n] \geq 0 \Rightarrow [n] + [\frac{1}{2} - n] \geq 0 \Rightarrow [n] + \frac{1}{2} - n \geq 0 \Rightarrow [n] \geq n - \frac{1}{2}$

$y. \frac{\cot t + 1}{\tan t + 1} = \frac{\frac{\cos t}{\sin t} + 1}{\frac{\sin t}{\cos t} + 1}$
 $\frac{\cos t + \sin t}{\sin t + \cos t} = 1$

$A = \left\{ k\frac{\pi}{4}, k\frac{\pi}{4} + \frac{\pi}{4} \right\}$

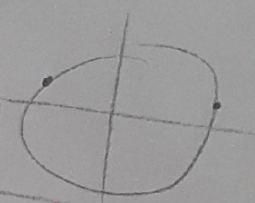


128

میانگین با 2ka یا 2ka+pi است؟

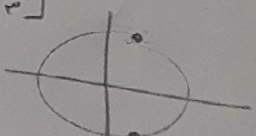
$y. \sqrt{1 - \cos^2 n}$
 $1 - \cos^2 n \geq 0 \Rightarrow \cos^2 n \leq 1 \Rightarrow \sin^2 n \leq 1 \Rightarrow \sin n \leq \frac{1}{2} \Rightarrow \left[-\frac{1}{2}, \frac{1}{2} \right]$

$y. \sqrt{1 - 2\cos n}$
 $1 - 2\cos n \geq 0 \Rightarrow 2\cos n \leq 1 \Rightarrow \cos n \leq \frac{1}{2}$



128

$y. \sqrt{1 - 2\cos m}$
 $1 - 2\cos m \geq 0 \Rightarrow 2\cos m \leq 1 \Rightarrow \cos m \leq \frac{1}{2}$



$y. \sqrt{1 - 2\cos(m^2 + m + 1)}$
 $1 - 2\cos(m^2 + m + 1) \geq 0 \Rightarrow \cos(m^2 + m + 1) \leq \frac{1}{2}$

باید بین جواب ها اشتراک بگیری!

$y. \sqrt{1 - 2\cos(m^2 + m + 1)}$
 $1 - 2\cos(m^2 + m + 1) \geq 0 \Rightarrow \cos(m^2 + m + 1) \leq \frac{1}{2}$

128

$y. \sqrt{1 - 2\cos(m^2 + m + 1)}$
 $1 - 2\cos(m^2 + m + 1) \geq 0 \Rightarrow \cos(m^2 + m + 1) \leq \frac{1}{2}$

$y. \log(1 - \log n)$
 $1 - \log n > 0 \Rightarrow \log n < 1 \Rightarrow n < 10$

از کجا آمد؟

128

$y. \log(1 - \log n)$
 $1 - \log n > 0 \Rightarrow \log n < 1 \Rightarrow n < 10$

$y. \sqrt{m - m^2}$
 $m - m^2 \geq 0 \Rightarrow m(1 - m) \geq 0 \Rightarrow 0 \leq m \leq 1$

128

$y. \left| \frac{m + w}{m + e} \right|$
 $\frac{m + w}{m + e} \in W \Rightarrow m \in \frac{m + w}{m + e} \cdot W$

۳- کمترین عبارت مفرج را دستکاری میکنیم تا ک ساده تر شود...

$$(x^2 - x_{n+1}) - 5|x-1| + 4 \rightarrow (x-1)^2 - 5|x-1| + 4 \xrightarrow{|x-1|=t} t^2 - 5t + 4$$

$$(t-1)(t-4) = 0 \rightarrow t=1, t=4$$

$$\rightarrow Df = \mathbb{R} - \{-3, 0, 2, 5\}$$

$$|x-1|=1 \rightarrow x=2, 0 \quad |x-1|=4 \rightarrow x=5, -3$$

$$\omega - \text{الف}) \left\lfloor x - \frac{1}{x} \right\rfloor + \left\lfloor x + \frac{2}{x} \right\rfloor \geq 0 \xrightarrow{+1} \left\lfloor x - \frac{1}{x} \right\rfloor + 1 + \left\lfloor x + \frac{2}{x} \right\rfloor \geq 1$$

$$\left\lfloor x + \frac{2}{x} \right\rfloor + \left\lfloor x + \frac{2}{x} \right\rfloor \geq 1 \rightarrow \left\lfloor x + \frac{2}{x} \right\rfloor \geq \frac{1}{2} \xrightarrow{\text{اولین عدد صحیح بزرگتر از } \frac{1}{2} \text{ برابر یک است!}} x + \frac{2}{x} \geq 1$$

$$x \geq \frac{1}{x} \rightarrow Df = \left[\frac{1}{x}, +\infty \right)$$

$$\omega - \text{ب}) \left\lfloor x - \frac{1}{x} \right\rfloor + \left\lfloor -x + \frac{1}{x} \right\rfloor \geq 0$$

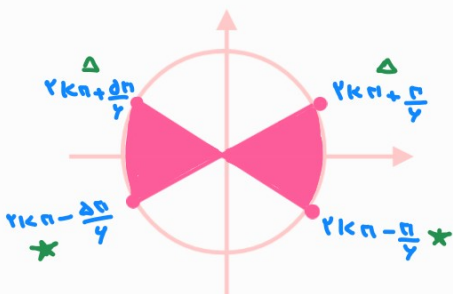
مردانیم:

$$\left\lfloor x \right\rfloor + \left\lfloor -x \right\rfloor \begin{cases} \hookrightarrow n \in \mathbb{Z} & 0 \checkmark \\ \hookrightarrow x \notin \mathbb{Z} & -1 \times \end{cases}$$

$$\rightarrow \left\lfloor x - \frac{1}{x} \right\rfloor \in \mathbb{Z} \rightarrow x \in \mathbb{Z} + \frac{1}{x}$$

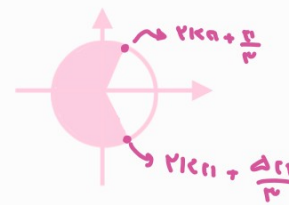
$$Df = \left\{ x \mid x = k + \frac{1}{x}, k \in \mathbb{Z} \right\}$$

$$4 - \text{ب}) 1 - 4 \sin^2 n \geq 0 \rightarrow 4 \sin^2 n \leq 1 \rightarrow \sin^2 n \leq \frac{1}{4} \rightarrow -\frac{1}{2} \leq \sin n \leq \frac{1}{2}$$



$$\rightarrow Df = \left[k\pi - \frac{\pi}{6}, k\pi + \frac{\pi}{6} \right]$$

ب- ٧) $1 - 2c \sin \theta > 0 \rightarrow \sin \theta \leq \frac{1}{r} \rightarrow Df_c \left[r \left(\pi + \frac{\pi}{r} \right), r \left(\pi + \frac{2\pi}{r} \right) \right]$



الف- ٨) $-1 \leq x^r + \frac{1}{x^r} \leq 1 \rightarrow -1 \leq x^r + \frac{1}{x^r} + \frac{q}{\varepsilon} - \frac{q}{\varepsilon} \leq 1$

$\frac{1}{\varepsilon} \leq x^r + \frac{1}{x^r} + \frac{q}{\varepsilon} \leq \frac{q}{\varepsilon} \rightarrow \frac{1}{\varepsilon} \leq \left(x + \frac{r}{x} \right)^r \leq \frac{q}{\varepsilon}$

$\left(x + \frac{r}{x} \right)^r \leq \frac{q}{\varepsilon} \rightarrow -\frac{r}{x} \leq x + \frac{r}{x} \leq \frac{r}{x} \rightarrow -r \leq x \leq r \quad I$

$\left(x + \frac{r}{x} \right)^r \geq \frac{1}{\varepsilon} \rightarrow x + \frac{r}{x} \geq \frac{1}{r} \rightarrow x \geq -1 \quad \text{و} \quad x + \frac{r}{x} \leq -\frac{1}{r} \rightarrow x \leq -r \quad II$

$I \cap II \rightarrow Df = [-r, -r] \cup [-1, 0]$

ب- ٨) $\sqrt{\omega - 2n} \rightarrow \omega - 2n \geq 0 \rightarrow n \leq \frac{\omega}{2} \rightarrow Df_c \left[r, \frac{\omega}{r} \right]$

$-1 \leq \sqrt{\omega - 2n} \leq 1 \rightarrow \omega - 2n \leq 1 \rightarrow -2n \leq -\omega + 1 \rightarrow n \geq r$

مجموعه برقرار است ✓

الف- ٩) $y = \log_r (1 - \log_r^n)$ $\rightarrow \log_r^n \rightarrow n > 0$

$\rightarrow 1 - \log_r^n > 0 \rightarrow \log_r^n < 1 \rightarrow n < r$

$Df_c (0, r)$

ب- ٩) $y = \log_r (1 - \log_r^n \frac{1}{r})$ $\rightarrow \log_r^n \frac{1}{r} \rightarrow n > 0$

$\rightarrow 1 - \log_r^n \frac{1}{r} > 0 \rightarrow \log_r^n \frac{1}{r} < 1 \rightarrow n > \frac{1}{r}$

منابع (١، ٢)

$Df_c \left(\frac{1}{r}, +\infty \right)$