

الف) $y = \sqrt{\frac{n-1}{n} - \frac{1}{n-1}} = y = \sqrt{\frac{(n-1)^2 - n^2}{n(n-1)}} = \sqrt{\frac{-2n+1}{n(n-1)}}$

$D = (-\infty, 0) \cup [\frac{1}{4}, 1)$

ب) $y = \frac{\frac{1}{n+1} - \frac{1}{n}}{\frac{1}{n-1} + \frac{1}{n+1}} = \frac{\frac{n+1-n}{n(n+1)}}{\frac{n+1+n+1}{(n-1)(n+1)}} = \frac{n+1-n}{n(n+1)} \cdot \frac{(n-1)(n+1)}{2n+2} = \frac{n-1}{2(n+1)}$

$D = \mathbb{R} - \{-1, 0, -\frac{1}{2}, -\frac{1}{3}, \dots\}$

ج) $f(n) = \sqrt{\left(\left(\frac{1}{p}\right)^n - q\right) \left(\frac{1}{p} - n^p\right)}$

$D = (-\infty, 1] \cup \left[\frac{1}{p}, +\infty\right)$

د) $\sqrt{n-1} + \sqrt{y+1} = \sqrt{y+1} = \sqrt{n+1} \Rightarrow \sqrt{n-1} = \sqrt{n+1} \Rightarrow n-1 = n+1 \Rightarrow -2 = 0$ (no solution)

$D = [1, 1]$

ه) $y = \frac{\log_p}{\sqrt{n^p-1} + 1}$

$D = (-\infty, 1] \cup [1, +\infty)$

و) $\sqrt{n^p-1} \neq -1 \Rightarrow n^p-1 \neq 1 \Rightarrow n^p \neq 2 \Rightarrow n \neq \sqrt[p]{2}$

$D = \mathbb{R} - \{\sqrt[p]{2}\}$

ز) $y = \sqrt{-n^p + an + p}$

$D = [-p, b]$

$b + a = \frac{p}{n} - \frac{1}{p} = 1$

ح) $f(n) = \begin{cases} p n - p & n \geq 1 \\ p n + p & n < 1 \end{cases}$

ط) $g(n) = \sqrt{f(n) - n}$

$D = [1, +\infty) \cup [-p, +\infty) = D$

ث) $f(n) - n \geq 0$

$\begin{cases} n \geq 1 & p n - p - n \geq 0 \Rightarrow p n - n - p \geq 0 \Rightarrow n(p-1) \geq p \Rightarrow n \geq \frac{p}{p-1} \\ n < 1 & p n + p - n \geq 0 \Rightarrow p n - n + p \geq 0 \Rightarrow n(p-1) \geq -p \Rightarrow n \geq -\frac{p}{p-1} \end{cases}$

$$f(n) = P_n^P - P_n$$

$$f\left(\frac{\sin \frac{\pi}{\sqrt{P}+1} + \cos \frac{\pi}{\sqrt{P}+1}}{\frac{\sqrt{P}+1}{P}}\right) = P\left(\frac{P+\sqrt{P}}{P}\right)^P - P\left(\frac{P+\sqrt{P}}{P}\right)$$

$$f\left(\frac{P+\sqrt{P}}{P}\right) = P\left(\frac{P+\sqrt{P}}{P}\right)^P - P\left(\frac{P+\sqrt{P}}{P}\right) = \frac{P}{P} + P\sqrt{P} - P - P\sqrt{P} = -\frac{1}{P}$$

$$f(n) = \frac{n+1}{n-1}$$

$$f(f(a)) = a$$

$$a = P$$

$$a \in (P - \{1\})$$

$$\frac{\left(\frac{a+1}{a-1}\right) + 1}{\left(\frac{a+1}{a-1}\right) - 1} = a$$

$$a = \{x | x \in \mathbb{R}, x \neq 1\}$$

$$f(n) = n^P - P_n \quad g(n) = \sqrt{n^P - 1}$$

$$f(g(a)) = P$$

$$\Rightarrow g(a) = P \Rightarrow \sqrt{a^P - 1} = P$$

$$\Rightarrow a = \pm \sqrt[P]{P^2 + 1}$$

$$n^P - P_n = P$$

$$\Rightarrow n = P$$

$$f(n+P) = n^P - a n + P$$

$$f(-P) = P^P + P^0 + P = P^P$$

$$n+P = -P \Rightarrow n = -P$$

$$f(P) = 1 - a + P = P$$

$$n+P = P \Rightarrow n = 0$$

$$f(P) \times f(-P) = P^P \times P^P = (P^P)^2$$

$$f(P \cos^n \theta + P \cos \theta) = \tan^n \theta$$

$$f(P, \theta) = P = P$$

$$\Rightarrow \tan^n \theta + 1 = \frac{1}{\left(\frac{1}{P}\right)^P} = P \Rightarrow \tan^n \theta = P-1$$

$$P \cos^n \theta + P \cos \theta = P, \theta$$

$$P \cos^n \theta + P \cos \theta - P = 0$$

$$\cos^n \theta + \cos \theta - 1 = 0$$

$$\begin{cases} \cos \theta = -\frac{1}{P} \\ \cos \theta = \frac{1}{P} \end{cases}$$

$$\begin{cases} \cos \theta = \frac{1}{P} \\ \cos \theta = -\frac{1}{P} \end{cases}$$