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| نام و نام خانوادگی: .....<br>پاسخنامه تشریحی تکلیف شماره ..... کلاس: .....<br>۲۰. <u>انفینیمایی تواری</u>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |   |
| الف) $\rightarrow \frac{x-1}{x} - \frac{x}{x-1} \gg 0 \rightarrow \frac{x^2+1-2x-x^2}{x(x-1)} \gg 0 \rightarrow \frac{1-2x}{x(x-1)} \gg 0$<br>$\frac{1}{x-1} - \frac{1}{x} \gg 0 \Rightarrow D_f = (-\infty, 0) \cup [1, +\infty)$<br>$\rightarrow \begin{cases} x+1 \neq 0 \rightarrow x \neq -1 \\ x \neq 0 \\ x-1 \neq 0 \rightarrow x \neq 1 \\ x+2 \neq 0 \rightarrow x \neq -2 \end{cases} \Rightarrow D_f = \mathbb{R} - \{-1, 0, 1, -2\}$<br>$\frac{2}{x-1} + \frac{1}{x+2} \neq 0 \rightarrow \frac{2}{x-1} \neq -\frac{1}{x+2} \rightarrow 2x+6+1-x \rightarrow x+7 \neq 0 \rightarrow x \neq -7$ | ۱ |
| الف) $\rightarrow \sqrt{\left(\frac{1}{x}\right)^2 - 9} (2x-4x) \gg 0 \rightarrow \frac{-2}{\frac{1}{x} - 9} \gg 0 \Rightarrow D_f = (-\infty, -\frac{2}{9}] \cup [\frac{2}{9}, +\infty)$<br>$\rightarrow \begin{cases} y+1 = (2-\sqrt{x-1})^2 \rightarrow y = (2-\sqrt{x-1})^2 - 1 \rightarrow x \gg 1 \\ y+1 = 2 - \sqrt{x-1} \rightarrow 2 - \sqrt{x-1} \gg 0 \rightarrow \sqrt{x-1} \leq 2 \rightarrow x-1 \leq 4 \rightarrow x \leq 5 \end{cases} \Rightarrow D_f = [1, 5]$                                                                                                                            | ۲ |
| $f(x) = \frac{\log(x^2-x-2)}{\sqrt{x^2-1}+1}$<br>$\begin{cases} x^2-x-2 \gg 0 \rightarrow (x-2)(x+1) \gg 0 \\ x^2-1 \gg 0 \rightarrow x^2 \gg 1 \rightarrow x \gg 1, x \leq -1 \\ \sqrt{x^2-1}+1 \neq 0 \rightarrow \text{همیشه برقرار} \end{cases} \Rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$                                                                                                                                                                                                                                                                                                      | ۳ |
| چون عبارت زیر رادیکال باید نامنفی باشد و از آنجایی که جواب یک بازه است، متوجه می شویم که اعداد ۲- و ۱- ریشه های عبارتند. پس داریم:<br>$\xrightarrow{x=2} 2^2-2a-4=0 \rightarrow a=-\frac{1}{2}$<br>$\xrightarrow{x=-1} 1-\frac{1}{2}x-x^2=0 \rightarrow +2x^2+x-6=0 \rightarrow x = \frac{-1 \pm \sqrt{1+48}}{4} = -2, \frac{3}{2} \Rightarrow b = \frac{3}{2}$<br>$a+b = (-\frac{1}{2}) + \frac{3}{2} = 1$                                                                                                                                                                                                 | ۴ |
| $\rightarrow f(x) \gg x$<br>if $x \gg 1$ : $3x-2 \gg x \rightarrow x \gg 1$<br>if $x < -1$ : $2x+3 \gg x \rightarrow x \gg -3$<br>$\Rightarrow D_f = [-3, +\infty)$                                                                                                                                                                                                                                                                                                                                                                                                                                         | ۵ |

$$\sin \frac{\pi}{e} = \frac{\sqrt{e}}{e} \quad , \quad \cos 2\pi = 1 \quad \rightarrow \quad \sin \frac{\pi}{e} + \cos 2\pi = \frac{\sqrt{e}}{e} + 1$$

$$f(x) = \sqrt{x} - \frac{1}{x} = \sqrt{x} - \frac{1}{\sqrt{x}} = \sqrt{x} - \frac{1}{\sqrt{x}} = \sqrt{x} \left( \frac{\sqrt{x}}{\sqrt{x}} - \frac{1}{\sqrt{x}} \right) = \sqrt{x} \left( \frac{\sqrt{x}-1}{\sqrt{x}} \right)$$

$$= \sqrt{x} \left( \frac{\sqrt{x}-1}{\sqrt{x}} \right) = \sqrt{x} \left( -\frac{1}{\sqrt{x}} \right) = -1$$

6

$$f(a) = \frac{a+1}{a-1}$$

$$f(f(a)) = a \rightarrow \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = a \rightarrow \frac{\frac{a+1+a-1}{a-1}}{\frac{a+1-a+1}{a-1}} = a \rightarrow \frac{\frac{2a}{a-1}}{\frac{2}{a-1}} = a \rightarrow \frac{2a}{2} = a \rightarrow a \neq 1$$

$$\Rightarrow a \in \mathbb{R} - \{1\}$$

7

$$g(x) = \sqrt{x^2 - 1}$$

$$f(g(x)) = -x \rightarrow x^2 - \frac{1}{x} = -x \rightarrow x^2 - \frac{1}{x} + x = 0 \rightarrow (x-1)^2 = 0 \rightarrow x = 1$$

$$\rightarrow f(g(x)) = -x \rightarrow g(x) = \sqrt{x^2 - 1} = 1 \rightarrow x^2 - 1 = 1 \rightarrow x^2 = 2 \rightarrow x = \pm \sqrt{2}$$

8

$$f(x+c) = (x-c)(x-2) \xrightarrow{x+c=t \rightarrow x=t-c} f(t) = (t-c)(t-2)$$

$$f(1) = (1-c)(1-2) = (-1)(-1) = 1$$

$$f(-2) = (-2-c)(-2-2) = (-2-c)(-4) = 4(2+c) = 4 \times 2 = 8$$

$$\Rightarrow f(1) \times f(-2) = 1 \times 8 = 8$$

9

$$\sqrt{e} \cos \frac{\pi}{e} + \Delta \cos \pi = \frac{V}{e} \rightarrow \Delta \cos \frac{\pi}{e} + 1 \cos \pi = \frac{V}{e} \rightarrow \cos \frac{\pi}{e} = \frac{1}{\sqrt{e}}, -\frac{V}{e}$$

$$-1 \leq \cos \frac{\pi}{e} \leq 1, \cos \frac{\pi}{e} = \frac{1}{\sqrt{e}}$$

$$\tan \frac{\pi}{e} = \frac{\sin \frac{\pi}{e}}{\cos \frac{\pi}{e}} = \frac{1 - \cos \frac{\pi}{e}}{\cos \frac{\pi}{e}} = \frac{1 - \frac{1}{\sqrt{e}}}{\frac{1}{\sqrt{e}}} = \sqrt{e} - 1$$

$$\Rightarrow f(\frac{\pi}{e}) = \sqrt{e} - 1$$

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