

الف) $\rightarrow \frac{x-1}{x} - \frac{x}{x-1} \gg 0 \rightarrow \frac{x^2+1-2x-x^2}{x(x-1)} \gg 0 \rightarrow \frac{1-2x}{x(x-1)} \gg 0 \rightarrow \frac{1}{x-1} - \frac{2}{x} \gg 0 \Rightarrow D_f = (-\infty, 0) \cup [1, 1)$

ب) $\begin{cases} x+1 \neq 0 \rightarrow x \neq -1 \\ x \neq 0 \\ x-1 \neq 0 \rightarrow x \neq 1 \\ x+2 \neq 0 \rightarrow x \neq -2 \end{cases} \Rightarrow D_f = \mathbb{R} - \{-1, 0, 1, -2\}$

ج) $\frac{3}{x-1} + \frac{1}{x+2} \neq 0 \rightarrow \frac{3}{x-1} \neq -\frac{1}{x+2} \rightarrow 3x+6+1-x \rightarrow 2x+7 \neq 0 \rightarrow x \neq -\frac{7}{2}$

الف) $\rightarrow \sqrt{\left(\frac{1}{x}\right)^2 - 9} (2x-4x) \gg 0 \rightarrow \left(\frac{1}{x^2} - 9\right)(2x-4x) \gg 0 \rightarrow \frac{-2}{x^2} - \frac{6}{x} \gg 0 \Rightarrow D_f = (-\infty, -\frac{1}{3}] \cup [\frac{1}{3}, \infty)$

ب) $\sqrt{y+1} = 3 - \sqrt{x-1} \rightarrow y+1 = (3 - \sqrt{x-1})^2 \rightarrow y = (3 - \sqrt{x-1})^2 - 1 \rightarrow x \gg 1$

ج) $\sqrt{y+1} = 3 - \sqrt{x-1} \rightarrow 3 - \sqrt{x-1} \gg 0 \rightarrow \sqrt{x-1} \leq 3 \rightarrow x-1 \leq 9 \rightarrow x \leq 10$

د) $\Rightarrow D_f = [1, 10]$

الف) $f(x) = \frac{\log(x^2-2)}{\sqrt{x^2-1}+1} \begin{cases} x^2-2 \gg 0 \rightarrow (x-2)(x+2) \gg 0 \rightarrow \frac{-1}{x^2} - \frac{2}{x} \gg 0 \rightarrow \frac{x^2}{x^2-1} \gg 0 \rightarrow x \gg 1, x \leq -1 \\ x^2-1 \gg 0 \rightarrow x^2 \gg 1 \rightarrow x \gg 1, x \leq -1 \\ \sqrt{x^2-1}+1 \neq 0 \rightarrow \text{همیشه برقرار} \end{cases} \Rightarrow D_f = (-\infty, -1) \cup (2, \infty)$

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چون عبارت زیر را دیکال باید نامنفی باشد و از آنجایی که جواب یک بازه دارد پس است. متوجه می شویم که اعداد ۲- و ۱- ریشه های عبارتند. پس داریم:

الف) $\xrightarrow{x=2} 3-2a-4=0 \rightarrow a=-\frac{1}{2}$

ب) $\xrightarrow{x=-1} 3-\frac{1}{2}x-x^2=0 \rightarrow +2x^2+x-6=0 \rightarrow x = \frac{-1 \pm \sqrt{1+48}}{4} = -2, \frac{3}{2} \Rightarrow b = \frac{3}{2}$

ج) $a+b = (-\frac{1}{2}) + \frac{3}{2} = 1$

الف) $\rightarrow f(x) \gg x$

ب) if $x \gg 1$: $3x-2 \gg x \rightarrow x \gg 1$

ج) if $x < 1$: $2x+3 \gg x \rightarrow x \gg -3$

د) $\Rightarrow D_f = [-3, \infty)$

$$\sin \frac{\pi}{e} = \frac{\sqrt{e}}{e} \quad , \quad \cos 2\pi = 1 \quad \rightarrow \quad \sin \frac{\pi}{e} + \cos 2\pi = \frac{\sqrt{e}}{e} + 1$$

$$f(x) = x^e - e x = x(x^e - e) = x(x-1)^e - 1 = x \left(\left(\frac{\sqrt{e}}{e} \right)^e - 1 \right)$$

$$= x \left(\frac{e}{e} - 1 \right) = x \left(-\frac{1}{e} \right) = -\frac{1}{e}$$

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$$f(a) = \frac{a+1}{a-1}$$

$$f(f(a)) = a \rightarrow \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = a \rightarrow \frac{\frac{a+1+a-1}{a-1}}{\frac{a+1-a+1}{a-1}} = a \rightarrow \frac{\frac{2a}{a-1}}{\frac{2}{a-1}} = a \rightarrow a \neq 1$$

$$a \in \mathbb{R} - \{1\}$$

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$$g(x) = \sqrt{x^2 - 1}$$

$$f(g(x)) = -x \rightarrow x^2 - e x = -x \rightarrow x^2 - e x + x = 0 \rightarrow (x-2)^2 = 0 \rightarrow x = 2$$

$$\rightarrow f(g(x)) = -x \rightarrow g(x) = \sqrt{x^2 - 1} = 2 \rightarrow x^2 - 1 = 4 \rightarrow x^2 = 5 \rightarrow x = \pm \sqrt{5}$$

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$$f(x+c) = (x-c)(x-2) \xrightarrow{x+c=t \rightarrow x=t-c} f(t) = (t-4)(t-2)$$

$$f(4) = (4-4)(4-2) = (-2) \times (-1) = 2$$

$$f(-2) = (-2-4)(-2-2) = (-6) \times (-4) = 24$$

$$\Rightarrow f(4) \times f(-2) = 2 \times 24 = 48$$

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$$r \cos^2 x + \Delta \cos x = \frac{V}{r} \rightarrow \Delta \cos^2 x + 1 \cos x - \frac{V}{r} = 0 \rightarrow \cos x = \frac{1}{r}, -\frac{V}{r}$$

$$-1 \leq \cos x \leq 1, \cos = \frac{1}{r}$$

$$\tan^2 = \frac{\sin^2}{\cos^2} = \frac{1 - \cos^2}{\cos^2} = \frac{1 - \frac{1}{r^2}}{\frac{1}{r^2}} = r^2$$

$$\Rightarrow f(r, \Delta) = r^2$$

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