

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} = \sqrt{\frac{-x+1}{x(x-1)}}$ دقت کن!
 $DP = (-\infty, 0) \cup (\frac{1}{x}, 1)$
 $D_f = (0, 1) \cup (1, +\infty)$

ب) $f(x) = \frac{\frac{1}{x+1} - \frac{1}{x}}{\frac{1}{x-1} + \frac{1}{x+1}} \rightarrow \frac{x(x+1)}{(x-1)(x+1)}$
 $D_f = \mathbb{R} - \{-1, 1, \frac{\omega}{4}\}$ ۱, ۵۰

الف) $f(x) = \sqrt{\left(\left(\frac{1}{x}\right)^2 - 9\right)(32 - x^2)}$
 $\frac{-2}{0} - \frac{2/\omega}{0} +$
 $D_f = \mathbb{R} - (-\infty, 2/\omega)$

ب) $\sqrt{x-1} + \sqrt{y+1} = 3$
 If $y = -1 \Rightarrow \sqrt{x-1} = 3 \Rightarrow x = 10$
 If $y = 1 \Rightarrow \sqrt{x-1} = 0 \Rightarrow x = 1$
 $D_f = [1, 10]$ ۲

$f(x) = \frac{\log(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1} \rightarrow x^2 - x - 2 = (x-2)(x+1) > 0$
 $\frac{-1}{0} - \frac{2}{0} +$
 $\sqrt{(x+1)(x-1)} \rightarrow \frac{-1}{0} - \frac{1}{0} +$
 $D_f = (-\infty, -1) \cup (2, +\infty)$ ۳

$f(x) = \sqrt{r + ax - x^2}$
 $D_f = [-r, b]$
 $-(x+r)(x-b) = -x^2 + (b-r)x + rb = -x^2 + ax + r$
 $b = \frac{r}{a} \quad a = -\frac{1}{r}$
 $a+b = \frac{r}{r} - \frac{1}{r} = 1$ ۴

$f(x) = \begin{cases} rx - r & x \geq 1 \\ rx + 3 & x < 1 \end{cases}$
 $rx - r - x \geq 0 \Rightarrow rx \geq r \Rightarrow x \geq 1 \checkmark$
 $rx + 3 - x \geq 0 \Rightarrow x \geq -3 \checkmark$
 $g(x) = \sqrt{f(x) - x}$
 $D_g = (-\infty, -3] \cup [1, +\infty)$
 $D_g = [-3, +\infty)$ ۵

$$f(x) = rx^r - x = rx(x-r)$$

$$f\left(\sin \frac{\pi}{r} + \cos r\pi\right) = \frac{r+\sqrt{r}}{r}$$

$$\downarrow \frac{\sqrt{r}}{r}$$

↓ 1

$$(r+\sqrt{r})\left(\frac{\sqrt{r}-r}{r}\right) = \frac{r-r}{r} = \boxed{-\frac{1}{r}}$$

✓

8

$$f(x) = \frac{x+1}{x-1} \quad x \neq 1 \Rightarrow a \neq 1$$

$$f(f(a)) = a$$

$$f\left(\frac{a+1}{a-1}\right) = a$$

$$a = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{\frac{r+1}{a-1}}{\frac{r}{a-1}} = \frac{ra}{r} = a = a \quad \checkmark$$

$$\rightarrow \mathbb{R} - \{1\}$$

✓

9

$$f(x) = x^r - x$$

$$g(x) = \sqrt{x^r - 1}$$

$$f(g(x)) = -x$$

$$x^r - x = -x \Rightarrow x^r - x + x = 0 \Rightarrow (x-r)^r = 0 \Rightarrow x = r = g(a)$$

$$\sqrt{x^r - 1} = r \Rightarrow x^r - 1 = r \Rightarrow x^r = r+1 \Rightarrow x = \sqrt[r]{r+1} = a$$

10

$$f(x+r) = x^r - \omega x + 9 \Rightarrow f(x) = (x-r)^r - \omega(x-r) + 9 = x^r - 9x + 9 - \omega x + 10 + 9$$

$$\Rightarrow f(x) = x^r - 11x + 28$$

$$f(-1) \times f(1) = \sqrt{1 \times 1} = \boxed{1}$$

✓

11

$$f(r \cos^r x + \omega \cos x) = \tan^r x$$

$$r \cos^r x + \omega \cos x = r/\omega \Rightarrow r \cos^r x + \omega \cos x - r/\omega = 0$$

$$\cos^r x + \omega \cos x - 1/r = 0 = (\cos - r)(\cos + r)$$

$$\cos x = \frac{r}{\omega} \text{ or } -\frac{r}{\omega}$$

$$\Rightarrow x = \frac{\pi}{r}, \frac{\omega\pi}{r}, \dots$$

$$f(r/\omega) = \tan^r\left(\frac{\pi}{r}\right) = \tan^r\left(\frac{\omega\pi}{\omega}\right) = \boxed{r}$$

✓

12

(الف-۱) $\frac{x-1}{x} - \frac{x}{x-1} \geq 0 \rightarrow \frac{(x-1)^2 - x^2}{x(x-1)} \geq 0 \rightarrow \frac{-x+1}{x(x-1)} \geq 0$

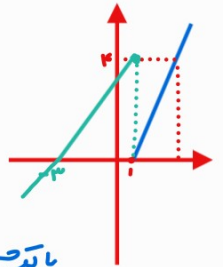
$\frac{1}{x} - \frac{1}{x-1} \geq 0 \rightarrow D_f = (-\infty, 0) \cup [\frac{1}{x}, 1)$

$$f(n) - n = \begin{cases} 3n - 2 - n & n \geq 1 \\ n + 3 - n & n < 1 \end{cases} = \begin{cases} 2n - 2 & n \geq 1 \\ n + 3 & n < 1 \end{cases}$$

$$f(n) - n \geq 0 \rightarrow \begin{cases} 2n - 2 \geq 0 \rightarrow n \geq 1 \checkmark \\ n + 3 \geq 0 \xrightarrow{n < 1} -3 \leq n < 1 \checkmark \end{cases}$$

$$\rightarrow D_f = [-3, +\infty)$$

با توجه به شکل نیز مشخص است!



-۵