

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} = \sqrt{\frac{-x^2+1}{x(x-1)}}$

$\frac{-1}{0} \quad \frac{1}{0} \quad \frac{1}{0}$

$D_f = (0, \frac{1}{2}] \cup (1, +\infty)$

ب) $f(x) = \frac{\frac{1}{x+1} - \frac{2}{x}}{\frac{2}{x-1} + \frac{1}{x+2}} \rightarrow \frac{x+2}{(x-1)(x+2)}$

$D_f = \mathbb{R} - \left\{ -2, -1, 0, 1, -\frac{2}{3} \right\}$

الف) $f(x) = \sqrt{\left(\left(\frac{1}{x}\right)^2 - 9\right)(x^2 - x^2)}$

$\frac{-2}{0} \quad \frac{2/0}{0}$

$D_f = \mathbb{R} - (-2, 2/0)$

ب) $\sqrt{x-1} + \sqrt{y+1} = 2$

If $y = -1 \Rightarrow \sqrt{x-1} = 2 \Rightarrow x = 5$

If $y = 1 \Rightarrow \sqrt{x-1} = 0 \Rightarrow x = 1$

$D_f = [1, 5]$

$f(x) = \frac{\log(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1} \rightarrow x^2 - x - 2 = (x-2)(x+1) > 0$

$\frac{-1}{0} \quad \frac{2}{0}$

$\sqrt{(x+1)(x-1)} \rightarrow \frac{-1}{0} \quad \frac{1}{0}$

$D_f = (-\infty, -1) \cup (2, +\infty)$

$f(x) = \sqrt{r+ax-x^2}$

$D_f = [-r, b]$

$-(x+r)(x-b) = -x^2 + (b-r)x + rb = -x^2 + ax + r$

$b = \frac{r}{a} \quad a = -\frac{1}{r}$

$a+b = \frac{r}{r} - \frac{1}{r} = 1$

$f(x) = \begin{cases} 2x-2 & x \geq 1 \\ 2x+2 & x < 1 \end{cases}$

$2x-2 \geq 0 \Rightarrow 2x \geq 2 \Rightarrow x \geq 1 \checkmark$

$2x+2 \geq 0 \Rightarrow x \geq -1 \checkmark$

$g(x) = \sqrt{f(x) - x}$

$D_g = (-\infty, -2] \cup [1, +\infty)$

$$f(x) = rx^r - x = rx(x-r)$$

$$f\left(\sin \frac{\pi}{r} + \cos r\pi\right) = \frac{r+\sqrt{r}}{r}$$

$$\downarrow \frac{\sqrt{r}}{r}$$

$$\downarrow 1$$

$$(r+\sqrt{r})\left(\frac{\sqrt{r}-r}{r}\right) = \frac{r-r}{r} = \boxed{-\frac{1}{r}}$$

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$$f(x) = \frac{x+1}{x-1} \quad x \neq 1 \Rightarrow a \neq 1$$

$$f(f(a)) = a$$

$$\rightarrow \mathbb{R} - \{1\}$$

$$f\left(\frac{a+1}{a-1}\right) = a$$

$$a = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{\frac{r+1}{a-1}}{\frac{r}{a-1}} = \frac{ra}{r} = a = a \checkmark$$

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$$f(x) = x^r - x$$

$$g(x) = \sqrt{x^r - 1}$$

$$f(g(x)) = -x$$

$$x^r - x = -x \Rightarrow x^r - x + x = 0 \Rightarrow (x-r)^r = 0 \Rightarrow x = r = g(a)$$

$$\sqrt{x^r - 1} = r \Rightarrow x^r - 1 = r \Rightarrow x^r = r+1 \Rightarrow x = \sqrt[r]{r+1} = a$$

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$$f(x+r) = x^r - \omega x + 9 \Rightarrow f(x) = (x-r)^r - \omega(x-r) + 9 = x^r - 9x + 9 - \omega x + 10 + 9$$

$$\Rightarrow f(x) = x^r - 11x + 28$$

$$f(-1) \times f(1) = \sqrt{1 \times 1} = \boxed{1}$$

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$$f(r \cos^r x + \omega \cos x) = \tan^r x$$

$$r \cos^r x + \omega \cos x = r/\omega \Rightarrow r \cos^r x + \omega \cos x - r/\omega = 0$$

$$\cos^r x + \omega \cos x - 1/r = 0 = (\cos - r)(\cos + r)$$

$$\cos x = \frac{r}{\omega} \text{ or } -\frac{r}{\omega}$$

$$\Rightarrow x = \frac{r}{\omega}, \frac{\omega}{r}, \dots$$

$$f(r/\omega) = \tan^r\left(\frac{\pi}{r}\right) = \tan^r\left(\frac{\omega\pi}{r}\right) = \boxed{r}$$

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