

نام و نام خانوادگی: راستینی ترکیبی یا سمنانه شریعی تکلیف شماره ۵ کلاس یازدهم پسر A

$$1) \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \Rightarrow \sqrt{\frac{x^2 - 2x + 1 - x^2}{x^2 - x}} \Rightarrow \sqrt{\frac{-2x+1}{x(x-1)}} \Rightarrow \text{[scribbled out]}$$

$$D_f = (-\infty, 0) \cup \left[\frac{1}{2}, 1\right)$$

۲) $f(x) = \frac{\frac{1}{x+1} - \frac{2}{x}}{\frac{2}{x-1} + \frac{1}{x+2}}$ ریشه یابی $\Rightarrow \{ -1, 0, 1, -2 \} \Rightarrow D_f = \mathbb{R} - \{ 0, 1, -1, -2 \}$

۳) $f(x) = \sqrt{\left(\left(\frac{1}{x}\right)^x - 9\right)(x^2 - 4^x)}$ $\left(\frac{1}{x}\right)^x = 9 \Rightarrow x = -2$ $x^2 - 4^x = 0 \Rightarrow 4^x = x^2 \Rightarrow x = 2, 5$

$$D_f = (-\infty, -2] \cup [2, 5, +\infty)$$

۴) $\sqrt{x-1} + \sqrt{y+1} = 2$ $x_{\max} = y = -1 \Rightarrow \sqrt{y+1} = 0 \Rightarrow \sqrt{x-1} = 2 \Rightarrow x = 5$

۵) $f(x) = \frac{\log_2(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$ $D_f = (1, 5]$ $\left. \begin{array}{l} x^2 - x - 2 > 0 \Rightarrow x < -1 \text{ or } x > 2 \\ x^2 - 1 \geq 0 \Rightarrow x \leq -1 \text{ or } x \geq 1 \end{array} \right\} \Rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$

۶) $f(x) = \sqrt{-x^2 + ax + 2}$ $D_f = [-2, b]$

$x = -2 \Rightarrow (-2)^2 + a(-2) + 2 = 0 \Rightarrow -2a = 0 \Rightarrow a = 0 \Rightarrow f(x) = \sqrt{-x^2 + 2}$

$-x^2 + \frac{1}{2}x + 2 = 0 \Rightarrow \frac{c}{a} = -2 \Rightarrow -2 \times b = -2 \Rightarrow b = 1 \Rightarrow a + b = \frac{1}{2} - \frac{1}{2} = 0$

۷) $g(x) = \sqrt{f(x) - x}$ $f(x) = \begin{cases} x^2 - 2 & x \geq 1 \\ x^2 + 2 & x < 1 \end{cases} \Rightarrow f(x) - x \geq 0 \Rightarrow$

$x \geq 1 \Rightarrow x^2 - 2 \geq x \Rightarrow x^2 - x - 2 \geq 0 \Rightarrow x \geq 2 \text{ or } x \leq -1$

$x < 1 \Rightarrow x^2 + 2 \geq x \Rightarrow (x \geq -2) \cap (x < 1) \Rightarrow -2 \leq x < 1 \Rightarrow D_f = [-2, +\infty)$

$f(x) = 2x^{\frac{1}{2}} - 4^x \Rightarrow f\left(\sin \frac{\pi}{6} + \cos \frac{\pi}{6}\right) \Rightarrow f\left(\frac{\sqrt{3}}{2} + 1\right) = f\left(\frac{\sqrt{3}+2}{2}\right) =$

$2\left(\frac{\sqrt{3}+2}{2}\right)^{\frac{1}{2}} - 4^{\frac{\sqrt{3}+2}{2}} = \frac{\sqrt{3}+2}{\sqrt{2}} - 2^{\sqrt{3}+2} \Rightarrow \frac{\sqrt{3}+2}{\sqrt{2}} - 2^{\sqrt{3}+2}$

۸) $f(x) = \frac{x+1}{x-1}$ $f(f(a)) = a$ $f(a) = \frac{a+1}{a-1} \Rightarrow a = f(f(a)) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} \Rightarrow$

$\frac{a+1+a-1}{a-1} \Rightarrow \frac{2a}{a-1} \Rightarrow \frac{2a}{a-1} = a \Rightarrow$

$a \neq 1 \Rightarrow D_f = \mathbb{R} - \{1\}$

$$f(x) = x^2 - 4x \quad g(x) = \sqrt{x^2 - 1} \quad f(g(a)) = -4$$

$$f(g(a)) = (\sqrt{a^2 - 1})^2 - 4\sqrt{a^2 - 1} = -4 \xrightarrow{\sqrt{a^2 - 1} = t} t^2 - 4t + 4 = 0 \Rightarrow (t - 2)^2 = 0$$

$$t = 2 \Rightarrow \sqrt{a^2 - 1} = 2 \Rightarrow a^2 - 1 = 4 \Rightarrow a^2 = 5 \Rightarrow \boxed{a = \pm \sqrt{5}}$$

$$f(x + \sqrt{5}) = x^2 - 5x + 9$$

$$x + \sqrt{5} = 4 \Rightarrow x = 4 - \sqrt{5} \Rightarrow f(4 - \sqrt{5}) = (4 - \sqrt{5})^2 - 5(4 - \sqrt{5}) + 9 = 2$$

$$x + \sqrt{5} = -4 \Rightarrow x = -4 - \sqrt{5} \Rightarrow f(-4 - \sqrt{5}) = (-4 - \sqrt{5})^2 - 5(-4 - \sqrt{5}) + 9 = 12$$

$$f(4 \cos^2 x + 5 \cos x) = \tan^2 x \Rightarrow 4 \cos^2 x + 5 \cos x = \frac{1}{\cos^2 x} \xrightarrow{\cos x = t} 4t^2 + 5t = \frac{1}{t^2} \Rightarrow 4t^4 + 5t^3 - 1 = 0$$

$$-1 \leq \cos x \leq 1 \Rightarrow -1 \leq t \leq 1 \Rightarrow \frac{1}{t^2} \leq 5 \Rightarrow \frac{1}{t^2} \leq 5 \Rightarrow \frac{1}{5} \leq t^2 \leq 1 \Rightarrow \frac{1}{\sqrt{5}} \leq |t| \leq 1$$

$$\cos x = \frac{1}{\sqrt{5}} \quad \sin^2 x + \cos^2 x = 1 \Rightarrow \sin^2 x + \frac{1}{5} = 1 \Rightarrow \sin^2 x = \frac{4}{5} \Rightarrow \sin x = \pm \frac{2}{\sqrt{5}}$$

$$\sin x = \frac{2}{\sqrt{5}} \Rightarrow \tan x = \frac{\frac{2}{\sqrt{5}}}{\frac{1}{\sqrt{5}}} = \boxed{2}$$