

$$\text{الف)} \frac{x-1}{x} - \frac{x}{x-1} \geq 0 \rightarrow \frac{(x-1)^2 - x^2}{x(x-1)} \geq 0 \rightarrow \frac{x^2 - 2x + 1 - x^2}{x(x-1)} \geq 0 \rightarrow \frac{-2x + 1}{x(x-1)} \geq 0$$

$$\Rightarrow D_f = (-\infty, 0) \cup \left(\frac{1}{2}, 1\right)$$

$$\text{ب)} \textcircled{1} x+1 \neq 0 \Rightarrow x \neq -1$$

$$\textcircled{2} x \neq 0$$

$$x-1 \neq 0 \Rightarrow x \neq 1$$

$$x+2 \neq 0 \Rightarrow x \neq -2$$

$$\textcircled{3} \frac{x}{x-1} \neq \frac{-1}{x+2} \Rightarrow x^2 + 2 \neq -x + 1 \rightarrow x^2 + x - 1 \neq 0 \Rightarrow x \neq \frac{-1 \pm \sqrt{5}}{2}$$

$$D_f = \mathbb{R} - \left\{-1, -\frac{1}{2}, 1, \frac{-1 \pm \sqrt{5}}{2}\right\}$$

$$\text{الف)} \left(\left(\frac{1}{x}\right)^x - 9\right) \cdot (2x - 2x) \geq 0 \rightarrow$$

$$x = -2$$

$$\rightarrow 2x - 2x = 0 \rightarrow 2^x = 2^x \Rightarrow x = 2, \omega$$

$$\frac{-2}{+} \frac{2, \omega}{+} \Rightarrow D_f = \mathbb{R} - (-2, 2, \omega)$$

$$\text{ب)} \textcircled{1} x-1 > 0 \rightarrow x > 1$$

$$\textcircled{2} \sqrt{x+1} = 3 - \sqrt{x-1} \Rightarrow 3 - \sqrt{x-1} \geq 0 \rightarrow x-1 \leq 11 \rightarrow x \leq 12$$

$$\textcircled{1} \cap \textcircled{2} \Rightarrow D_f = [1, 12]$$

$$\textcircled{1} \log_f(x^2 - x - 2) \rightarrow f \neq 1, f > 0, x^2 - x - 2 > 0 \rightarrow (x+2)(x-1) > 0 \rightarrow \frac{-1}{+} \frac{2}{-} \Rightarrow \mathbb{R} - [-1, 2]$$

$$\textcircled{2} x^2 - 1 \geq 0 \rightarrow x^2 \geq 1 \rightarrow x \geq 1, x \leq -1$$

$$\textcircled{3} \sqrt{x^2 - 1} \neq -1 \rightarrow \checkmark$$

$$\textcircled{1} \cap \textcircled{2} \cap \textcircled{3} \rightarrow D_f = \mathbb{R} - [-1, 2]$$

وقتی سیمی زیر منحنی است و بازه به شکل $[a, b]$ است، و $a < 0$ ، در نتیجه سیمی به شکل زیر است



$$x = -2 \Rightarrow 3 - 2a - 4 = 0 \Rightarrow -2a - 1 = 0 \rightarrow a = -\frac{1}{2}$$

$$x = b \Rightarrow 3 + \left(-\frac{1}{2}\right)b - b^2 = 0 \rightarrow b^2 + \frac{b}{2} - 3 = 0 \rightarrow 2b^2 + b - 6 = 0$$

$$ac \rightarrow b^2 + b - 12 = 0 \rightarrow (b+4)(b-3) = 0$$

$$\Rightarrow b = -4 \text{ یا } b = 3$$

$$\text{پس } b = 3 \text{ است}$$

$$f(x) \cdot x \geq 0 \rightarrow \begin{cases} x \geq 1 \rightarrow 2x - 2 - x \geq 0 \rightarrow x - 2 \geq 0 \rightarrow x \geq 2 \rightarrow [2, +\infty) \\ x < 1 \rightarrow 2x + 3 - x \geq 0 \rightarrow x + 3 \geq 0 \rightarrow x \geq -3 \rightarrow [-3, 1) \end{cases}$$

$$U \rightarrow [-3, 1) \cup [2, +\infty) = [-3, +\infty) = D_g$$

$$\left. \begin{array}{l} \sin \frac{\pi}{r} = \sin \pi = \sqrt{r}/r \\ \cos \pi = \cos \pi = -1 \end{array} \right\} f(\sin \frac{\pi}{r} + \cos \pi) = f(\frac{\sqrt{r}}{r})$$

6

$$\rightarrow f(x) = rx - f_r \xrightarrow{x = \frac{\sqrt{r}}{r}} f(\frac{\sqrt{r}}{r}) = \cancel{r(\frac{\sqrt{r}}{r}) - f_r} = \cancel{r - f_r} = \cancel{r - r} = 0$$

$$r(1 + \frac{\sqrt{r}}{r})^r - f(1 + \frac{\sqrt{r}}{r}) = -\frac{r}{r}$$

$$f(a) = \frac{a+1}{a-1} \rightarrow f(f(a)) = f(\frac{a+1}{a-1}) = a \rightarrow \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = a \rightarrow \frac{\frac{a+1+a-1}{a-1}}{\frac{a+1-a+1}{a-1}} = a$$

7

$$\rightarrow \frac{\frac{ra}{a-1}}{\frac{r}{a-1}} = a \rightarrow \cancel{\frac{ra}{a-1} \cdot \frac{a-1}{r}} = a \Rightarrow a \in \mathbb{R} - \{1\}$$

$$g(a) = \sqrt{a^r - 1} \rightarrow f(g(a)) = -f \rightarrow f(\sqrt{a^r - 1}) = -f \rightarrow (\sqrt{a^r - 1})^r - r\sqrt{a^r - 1} = -f$$

8

$$a^r - 1 - f\sqrt{a^r - 1} + f = 0 \xrightarrow{t = \sqrt{a^r - 1}} t^r - ft + f = 0 \rightarrow (t-r)^r = 0 \Rightarrow t = r$$

$$t = r \rightarrow \sqrt{a^r - 1} = r \rightarrow a^r - 1 = r^2 \rightarrow a^r = r^2 + 1 \Rightarrow a = \pm \sqrt{r^2 + 1}$$

$$f(r) = f(x+r) \Rightarrow x=1 \rightarrow f(r) = (1)^r \cdot a(1) + r = r$$

$$f(-r) = f(r+r) \Rightarrow x=-r \rightarrow f(-r) = (-r)^r - a(-r) + r = vr$$

9

$$f(r) \times f(-r) = 1 \neq f$$

$$f(r, a) = f(f \cos^r x + a \cos x) \rightarrow f \cos^r x + a \cos x - r, a = 0 \xrightarrow{t = \cos x} f t^r + a t - r, a = 0$$

$$\xrightarrow{ac} t^r + a t - 1f = 0 \rightarrow (t+r)(t-r) = 0 \rightarrow \cos x = -r, r \rightarrow \text{بفرض } r, \text{ جواب ندارد.}$$

10

$$f(\frac{r}{a}) = f(f \cos^r x + a \cos x) \rightarrow f \cos^r x + a \cos x - \frac{r}{a} = 0 \xrightarrow{t = \cos x} f t^r + a t - \frac{r}{a} = 0 \xrightarrow{x=a} \frac{1 - \frac{1}{r}}{\frac{1}{r}} = 1a$$

$$r \cdot t^r + r a t - r = 0 \rightarrow (t-1)(a t + r) = 0 \rightarrow \cos x = \frac{r}{a} \Rightarrow f(\frac{r}{a}) = \frac{1 - \frac{r}{a}}{\frac{r}{a}} = \frac{14}{9}$$