

الف) $\frac{(x-1)^2 - x^2}{x(x-1)} \geq 0 \quad \frac{x^2 - 2x + 1 - x^2}{x(x-1)} \geq 0 \quad \frac{1-2x}{x(x-1)} \geq 0$ $D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$ ب) $f(x) = \frac{-(x+2)(x-1)}{(x+1)(x+5)}$ $\rightarrow x+5 \neq 0 \rightarrow x \neq -5$ $D_f = (-\frac{5}{4}, -1) \cup (0, 1)$ باید تا عبارت‌هایی که در صورت‌مخرج هستند صاف و باقی‌مانده عبارت را ساده کنی!	۱
الف) $[\frac{1}{t^2} - 9](32 - t^2) \geq 0 \quad t = 2^x \rightarrow t > 0 \quad \frac{1}{t} - 9 = \frac{1-9t}{t} \rightarrow 32 - t^2$ $1-9t = 0 \rightarrow t = \frac{1}{9} \quad 32 - t^2 = 0 \rightarrow t = \pm\sqrt{32}$ $D_f = (-\infty, -\frac{1}{9}] \cup [\frac{\sqrt{2}}{2}, +\infty)$ ب) $3 - \sqrt{x-1} \geq 0 \quad \sqrt{x-1} \leq 3$ $D = [-\infty, 10]$ بانت نبوی!	۲
$x^2 - x - 2 > 0 \quad (x-2)(x+1) > 0$ $x < -1 \quad x > 2$ $D = (-\infty, -1) \cup (2, \infty)$	۳
$3 + ax - x^2 \geq 0 \quad [-2, b] \quad (-2)b = \frac{c}{A} = \frac{3}{-1} = -3 \quad b = \frac{3}{2}$ $-2 + b = a \quad a = -2 + \frac{3}{2} = -\frac{1}{2}$ $a + b = -\frac{1}{2} + \frac{3}{2} = 1$ جواب	۴
$f(x) = 3x - 2 \quad 3x - 2 - x \geq 0 \quad 2x - 2 \geq 0 \rightarrow x \geq 1$ $f(x) = 2x + 3 \quad 2x + 3 - x \geq 0 \quad x + 3 \geq 0 \rightarrow x \geq -3$ $[-3, 1) \cup [1, \infty) = [-3, \infty)$ جواب	۵

$f(x) = 2x^2 - 5x$ $\sin \frac{\pi}{4} + \cos 2\pi = \frac{\sqrt{2}}{2} + 1$ $f\left(\frac{\sqrt{2}}{2} + 1\right) = 2\left(1 + \frac{\sqrt{2}}{2}\right)^2 - f\left(1 + \frac{\sqrt{2}}{2}\right) = -\frac{1}{2}$	6
$f(x) = \frac{x+1}{x-1}$ $f(f(a)) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1}$ $\frac{a+1+a-1}{a-1} = \frac{2a}{a-1}$ $\frac{a+1-a+1}{a-1} = \frac{2}{a-1}$ $f(f(a)) = \frac{\frac{2a}{a-1}}{\frac{2}{a-1}} = a$ $a \neq 1$ $a \in \mathbb{R} - \{1\}$	7
$f(x) = x^2 - 5x$ $g(x) = \sqrt{x^2 - 1}$ $f(g(a)) = -4$ $(\sqrt{a^2 - 1})^2 - 5\sqrt{a^2 - 1} = -4$ $a^2 - 1 - 5\sqrt{a^2 - 1} = -4$ $t = \sqrt{a^2 - 1}$ $t^2 - 5t = -4$ $t^2 - 5t + 4 = 0$ $(t-2)(t-3) = 0$ $t = 2$ $\sqrt{a^2 - 1} = 2$ $a^2 - 1 = 4$ $a^2 = 5$ $a = \pm\sqrt{5}$	8
$f(x+2) = x^2 - 5x + 4$ $x+2=0 \rightarrow x=-2$ $f(2) = 1^2 - 5(1) + 4 = 0$ $x+2 = -2 \Rightarrow x = -4$ $f(-2) = (-4)^2 - 5(-4) + 4 = 16 + 20 + 4 = 40$ $f(2) \times f(-2) = 0 \times 40 = 0$	9
$3\cos^2 x + 2\cos x = 3, 0$ $\cos x = t$ $3t^2 + 2t = 3, 0$ $3t^2 + 2t - 3 = 0$ $(3t-1)(t+3) = 0$ $t = \frac{1}{3}$ $t = -3$ $\cos x = \frac{1}{3}$ $\tan^2 x = \frac{1 - \cos^2 x}{\cos^2 x} = \frac{1 - \frac{1}{9}}{\frac{1}{9}} = 8$ $\tan x = \pm 2\sqrt{2}$	10

تمام ریشه‌های مضرب‌ها نباید در دامنه باشند! $x+1 \neq 0 \rightarrow x \neq -1$ $x \neq 0$ $x-1 \neq 0 \rightarrow x \neq 1$

$$x+2 \neq 0 \rightarrow x \neq -2 \quad \therefore \quad \frac{x}{x-1} + \frac{1}{x+2} \neq 0 \rightarrow \frac{x^2+x+n-1}{(x-1)(x+2)} \neq 0 \rightarrow x^2+x \neq 0 \rightarrow x \neq -\frac{5}{2}$$

$$D_f = \mathbb{R} - \left\{ -1, 0, 1, -2, -\frac{5}{2} \right\}$$

(۲-الف) $\left(\left(\frac{1}{x} \right)^n - 9 \right) (3x - x^n) \geq 0 \rightarrow$

$\downarrow$
 $\left(\frac{1}{x} \right)^n = 9 \rightarrow x = -\frac{1}{3}$

$\downarrow$
 $3x = x^n \rightarrow x = \frac{5}{2}$

$\frac{-1}{+} \quad \frac{5}{-} \quad \frac{5}{+}$

$\rightarrow D_f = (-\infty, -\frac{1}{3}] \cup [\frac{5}{2}, +\infty)$

(۲-ب) $\sqrt{x-1} + \sqrt{y+1} = 3$, $\sqrt{y+1} = 3 - \sqrt{x-1} \rightarrow 3 - \sqrt{x-1} \geq 0 \rightarrow \sqrt{x-1} \leq 3 \rightarrow x \leq 10$

$\downarrow$
 $x-1 \geq 0 \rightarrow x \geq 1$

$\underbrace{\sqrt{y+1}}_{\text{همواره نامنفی!}}$

$I, II \rightarrow 1 \leq x \leq 10 \rightarrow D_f = [1, 10]$