

1. $x \neq 0$ و $x \neq 1$ مخرج مشترک $\frac{(x-1)^2 - x^2}{x(x-1)} \rightarrow \frac{x^2 - 2x + 1 - x^2}{x(x-1)} \rightarrow \frac{1-2x}{x(x-1)} \geq 0$

$x \neq 1$ و $x \neq -2$ و $x \neq -1$ و $x \neq 0$ $D = (-\infty, 0) \cup (\frac{1}{2}, 1)$ $x \geq \frac{1}{2}$

$\frac{x^2(x+2) + (x-1)}{(x-1)(x+2)} \rightarrow \frac{x^3 + 2x^2 + x - 1}{(x-1)(x+2)} \rightarrow D = R - \{0, -2, -1\}$

2. $(\frac{1}{x})^x - 1 = 0 \rightarrow x = -2$ $\begin{cases} 3x - 4x^2 = 0 \\ x = -2 \end{cases}$

$D = (-\infty, -2] \cup [0, -1)$

3. $\sqrt{x-1} + \sqrt{x+1} = 3 \rightarrow \sqrt{x+1} = 3 - \sqrt{x-1} \rightarrow x \geq 1 \rightarrow (3 - \sqrt{x-1})^2 \rightarrow 11 \geq x^2 - 6x + 2$

$11 \geq (x^2+1)(x^2-1) \rightarrow -4x \leq 4 \rightarrow D = (-4, 4] \cup [3, 3]$

4. $x \geq 1 \rightarrow \sqrt{3-2a} + 2 \geq 0 \rightarrow -2a + 4 \geq 0 \rightarrow -2a \geq -4 \rightarrow 2a \leq 4 \rightarrow a \leq \frac{4}{2}$

$ax^2 + bx + c \rightarrow -2b = \frac{3}{-1} = -3 \rightarrow 2b = 3 \rightarrow b = \frac{3}{2} \rightarrow \frac{3}{2} - 2 = a \rightarrow a = -\frac{1}{2}$

$x = -2 \rightarrow$ یکی از ریشه ها

$a + b = -\frac{1}{2} + \frac{3}{2} = \frac{2}{2} = 1$

5. $\log_{\frac{1}{2}}(x^2 - x - 2) > 0 \rightarrow (x-2)(x-1) > 0$ $\begin{cases} x \neq 1 \\ x \neq 2 \end{cases} \rightarrow (x-2)(x-1) > 1 \rightarrow R$

$D = R - \{1, 2\}$

6. $\sqrt{f(x) - x} \geq 0 \rightarrow 2x - 2 - x \geq 0 \rightarrow x - 2 \geq 0 \rightarrow x \geq 2$

$\sqrt{f(x) - x} \geq 0 \rightarrow 2x + 3 - x \geq 0 \rightarrow x + 3 \geq 0 \rightarrow x \geq -3$

$[-3, 2] \cup [1, +\infty)$

$$f\left(\sin \frac{\pi}{4} + \cos \frac{\pi}{4}\right) = \left(\frac{\sqrt{2}}{2} + 1\right) \left(\frac{\sqrt{2}}{2} + 1\right) = \left(\frac{\sqrt{2}}{2} + 1\right)^2 = f\left(\frac{\sqrt{2}}{2} + 1\right)$$

$$\left(\frac{\sqrt{2}}{2} + 1 + \sqrt{2}\right) \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2} + 1 + \sqrt{2} - \sqrt{2} - f = -1/2$$

$$f\left(\frac{\sqrt{2}}{2} + 1 + \sqrt{2}\right) = \sqrt{2} - f = -\frac{1}{2}$$

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$$f(f(a)) = \frac{x+1}{x-1} + 1 \rightarrow \frac{x+1+x-1}{x-1} = \frac{2x}{x-1} \rightarrow x \neq 1$$

$$x=a \quad \frac{x+1}{x-1} - 1 \rightarrow \frac{x+1-x+1}{x-1} = \frac{2}{x-1} = x$$

$$D = R - \{1\}$$

$$f(g(a)) = -f \rightarrow \tau^2 - f\tau = -f \rightarrow \tau^2 - f\tau + f = 0 \rightarrow (\tau - f)^2 = 0$$

$$\tau - f = 0 \rightarrow \tau = f$$

$$f(a)\sqrt{x^2-1} = 1 \rightarrow |x^2-1| = 1 \rightarrow x^2-1 = 1 \rightarrow x^2 = 2 \rightarrow x = \pm\sqrt{2}$$

$$x^2-1 = -1 \rightarrow x^2 = 0 \rightarrow x = 0$$

$$f(x+y) = x^2 - 2xy + y^2 \rightarrow f(4) = 1 - 4 + 4 = 0 \rightarrow 1 + 4 - 4 = 1$$

$$f(1) \times f(1) = 0$$

1/2

$$(f \cos^2 + a \cos) = \tan^2, \quad f \cos^2 + a \cos = 1/2 \rightarrow (f \cos^2 + a \cos - 1/2) = 0$$

$$\cos = \tau \quad \wedge \tau^2 + 10\tau - 1 = 0 \rightarrow (\tau - 1)(\tau + 1) = 0$$

$$-1 \leq \cos \leq 1 \quad \cos = \tau = \frac{1}{2} \quad \tau = \frac{1}{2} = -1/2$$

$$\sin^2 = 1 - \cos^2 \rightarrow 1 - \frac{1}{4} = \frac{3}{4} \rightarrow \tan^2 = \frac{3}{1} = 3$$

تمام ریشه های مقعرها نباید در دامنه باشند! $x+1 \neq 0 \rightarrow x \neq -1$ $x \neq 0$ $x-1 \neq 0 \rightarrow x \neq 1$

$$x+2 \neq 0 \rightarrow x \neq -2 \quad \therefore \quad \frac{x}{x-1} + \frac{1}{x+2} \neq 0 \rightarrow \frac{x^2+x+n-1}{(x-1)(x+2)} \neq 0 \rightarrow x^2+x \neq 0 \rightarrow x \neq -\frac{1}{2}$$

$$Df = \mathbb{R} - \left\{ -1, 0, 1, -2, -\frac{1}{2} \right\}$$

(۲-الف) $\left(\left(\frac{1}{x} \right)^n - 9 \right) (3x - \frac{1}{x}) \geq 0 \rightarrow \frac{-2}{+0} - \frac{0}{-0} + \frac{0}{+0} \rightarrow Df = (-\infty, -2] \cup [\frac{1}{3}, +\infty)$

$\left(\frac{1}{x} \right)^n = 9 \rightarrow x = -2$ $3x = \frac{1}{x} \rightarrow x = \frac{1}{3}$

(۲-ب) $\sqrt[n]{x-1} + \sqrt{y+1} = 3$, $\sqrt{y+1} = 3 - \sqrt[n]{x-1} \rightarrow 3 - \sqrt[n]{x-1} \geq 0 \rightarrow \sqrt[n]{x-1} \leq 3 \rightarrow x \leq 27$

$\downarrow$
 $x-1 \geq 0 \rightarrow x \geq 1$

$$I, II \rightarrow 1 \leq x \leq 27 \rightarrow Df = [1, 27]$$

$3 - \log_{\frac{1}{e}} x^2 - n - 2 \rightarrow x^2 - n - 2 > 0 \rightarrow (n-2)(n+1) > 0$ $\frac{-1}{+0} \frac{2}{-0} \rightarrow n > 2 \leq n < -1$ I

$\sqrt{x^2-1} + 1 \rightarrow$ همواره مثبت است $\rightarrow \sqrt{x^2-1} \rightarrow x^2-1 \geq 0 \rightarrow x \geq 1 \leq x \leq -1$ II

$$I, II \rightarrow x < -1, n > 2 \rightarrow Df = (-\infty, -1) \cup (2, +\infty)$$

$$f(n) = x^n - \varepsilon n = x^n - \varepsilon n + 2 - 2 = 2(x^n - \varepsilon n + 1) - 2 = 2(n-1)^2 - 2 \quad \underline{4}$$

$$\sin \frac{\pi}{x} + \cos \pi = \sqrt{\frac{x}{x}} + 1 \rightarrow f\left(\sqrt{\frac{x}{x}} + 1\right) = 2\left(\sqrt{\frac{x}{x}} + 1 - 1\right)^2 - 2 = 2\left(\frac{x}{x}\right) - 2 = -\frac{1}{x}$$

$$x+3=2 \rightarrow x=-1 \rightarrow f(\varepsilon) = 1^2 - \Delta + 4 = 2 \quad \underline{9}$$

$$x+3=-5 \rightarrow x=-4 \rightarrow f(-\varepsilon) = (-4)^2 - \Delta(-4) + 4 = 16 + 4 + 4 = 24$$

$$f(\varepsilon) \times f(-\varepsilon) = 2 \times 24 = 48$$