

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \geq 0 \Rightarrow \frac{x-1}{x} \geq \frac{x}{x-1} \Rightarrow (x-1)^2 \geq x^2 \Rightarrow x^2 - 2x + 1 \geq x^2$
 $-2x + 1 \geq 0 \Rightarrow x \leq \frac{1}{2}$
 $f(x) = \frac{1}{x+1} - \frac{x}{x-1}$
 $\frac{1}{x+1} - \frac{x}{x-1} = 0 \Rightarrow \frac{x-1-x(x+1)}{(x+1)(x-1)} = 0 \Rightarrow \frac{x-1-x^2-x}{(x+1)(x-1)} = 0 \Rightarrow \frac{-x^2-2x}{(x+1)(x-1)} = 0$
 $-x^2-2x = 0 \Rightarrow x(x+2) = 0 \Rightarrow x = 0, -2$
 $D_f = \mathbb{R} - \{0, -2\}$
 $x \neq \{1, 0, -1, -2\}$
 $\frac{x}{x-1} + \frac{1}{x+1} > 0 \Rightarrow \frac{x^2 + x + x - 1}{(x-1)(x+1)} > 0 \Rightarrow \frac{2x^2 - 1}{(x-1)(x+1)} > 0$
 $2x^2 - 1 > 0 \Rightarrow x^2 > \frac{1}{2} \Rightarrow x < -\frac{1}{\sqrt{2}} \text{ or } x > \frac{1}{\sqrt{2}}$
 $D_f = \mathbb{R} - \{0, -2, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\}$

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ب) $f(x) = \sqrt{((\frac{1}{x})^4 - 9)(3x^2 - 4x)} \geq 0 \Rightarrow 3x^2 - 4x \geq 0 \Rightarrow x(3x - 4) \geq 0 \Rightarrow x \leq 0 \text{ or } x \geq \frac{4}{3}$
 $(\frac{1}{x})^4 - 9 \geq 0 \Rightarrow \frac{1}{x^4} \geq 9 \Rightarrow x^4 \leq \frac{1}{9} \Rightarrow x \leq -\frac{1}{3} \text{ or } x \leq \frac{1}{3}$
 $D_f = (-\infty, -\frac{1}{3}] \cup [\frac{1}{3}, \frac{4}{3}] \cup [\frac{4}{3}, +\infty)$
 $\sqrt{x-1} + \sqrt{y+1} = 3 \Rightarrow \sqrt{y+1} = 3 - \sqrt{x-1} \Rightarrow y+1 = 9 - 6\sqrt{x-1} + x - 1 \Rightarrow y = x - 6\sqrt{x-1} + 7$
 $\sqrt{x-1} \geq 0 \Rightarrow x \geq 1$
 $\sqrt{y+1} \geq 0 \Rightarrow y \geq -1$
 $x-1 \leq 11 \Rightarrow x \leq 12$
 $D_f = [1, 12]$

ج) $f(x) = \frac{\log_f(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$
 $III \Rightarrow x^2 - x - 2 > 0 \Rightarrow (x-2)(x+1) > 0 \Rightarrow x < -1 \text{ or } x > 2$
 $I = \sqrt{x^2 - 1} \geq 0 \Rightarrow x^2 \geq 1 \Rightarrow x \geq 1 \text{ or } x \leq -1$
 $II = \sqrt{x^2 - 1} + 1 \neq 0 \Rightarrow \sqrt{x^2 - 1} \neq -1$
 $D_f = (-\infty, -1) \cup (2, +\infty)$

د) $\sqrt{1 + ax - x^2} - 2 \geq 0 \Rightarrow \sqrt{1 + ax - x^2} \geq 2 \Rightarrow 1 + ax - x^2 \geq 4 \Rightarrow -x^2 + ax - 3 \geq 0 \Rightarrow x^2 - ax + 3 \leq 0$
 $\Delta = a^2 - 12 \geq 0 \Rightarrow a \geq 2\sqrt{3} \text{ or } a \leq -2\sqrt{3}$
 $x = \frac{a \pm \sqrt{a^2 - 12}}{2}$

ه) $\Rightarrow -(\frac{x^2}{\frac{1}{x}} - \frac{x}{\frac{1}{x}} - 2) \Rightarrow (x - \frac{1}{x})(x + \frac{1}{x}) \Rightarrow [-2, \frac{3}{2}]$
 $\frac{a+b}{\frac{1}{x} + \frac{1}{x}} = 1$

و) $f(x) = \begin{cases} x^2 & x \geq 1 \\ 2x + 2 & x < 1 \end{cases}$
 $g(x) = \begin{cases} x^2 - 2 & x \geq 1 \\ x + 2 & x < 1 \end{cases}$
 $g(x) = \sqrt{f(x) - a}$
 $D_g = [-3, +\infty)$

$$f\left(\sin \frac{\pi}{r} + \cos r\pi\right) = \underbrace{\left(2\left(\frac{\sqrt{r}}{r}\right) + 2(1)\right)}_{\sqrt{r}+2} \underbrace{\left(\frac{\sqrt{r}}{r} + 1 - 2\right)}_{\frac{\sqrt{r}}{r}-1} \rightarrow \frac{r}{r} + \sqrt{r} - \sqrt{r} - 2$$

$$\sin \frac{\pi}{r} = \frac{\sqrt{r}}{r}$$

$$\cos r\pi = 1$$

$\frac{-1}{r}$

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$$f(a) = \frac{a+1}{a-1} \rightarrow f(f(a)) \rightarrow f\left(\frac{a+1}{a-1}\right) \rightarrow \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} \rightarrow \frac{a+1+a-1}{a+1-a-1} = \frac{2a}{a-2}$$

$$\frac{f(a)}{a} > a \rightarrow \checkmark \quad \left\{ R - \{1\} \right\}$$

$a \neq 1$

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$$g(a) = \sqrt{a^r - 1}$$

$$f(\sqrt{a^r - 1}) = \left(\sqrt{a^r - 1}\right)^r - f\sqrt{a^r - 1} = -f \rightarrow a^r - 1 - f\sqrt{a^r - 1} + f = 0$$

$x^r = a \rightarrow x = \pm \sqrt[r]{a}$

$$\left(\sqrt{a^r - 1} - 2\right)^r = 0 \rightarrow \sqrt{a^r - 1} = 2 \rightarrow a^r - 1 = 4 \rightarrow a^r = 5 \rightarrow a = \sqrt[r]{5}$$

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$$\left. \begin{aligned} f(1+r) &= 1 - 0 + 4 \rightarrow 5 \\ f(-4+r) &= r^2 + r + 4 \rightarrow \text{?} \end{aligned} \right\} f(r) \cdot f(-r) = \cancel{r^2} \rightarrow r^2$$

$r \times r = 144$

$! \text{ دقت } (4)^2 - 4(-4) + 4 = 4r$

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$$f(\tan a + \cot a) = \tan a \rightarrow \tan a \Rightarrow \tan \frac{\pi}{4} = 1$$

$$r_1 t + 0 + - r_2 0 = 0$$

$$11t + 10t - 4 = 0 \rightarrow (11t-4)(10t+4) = 0$$

$$(11t-4)(10t+4) \rightarrow t = \frac{4}{11}, t = -\frac{4}{10}$$

$\left\{ \begin{aligned} \cos a &= \frac{1}{r} \rightarrow \frac{r}{2}, \frac{\pi}{r} \end{aligned} \right.$

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$$(1) \frac{x-1}{x} - \frac{x}{x-1} \geq 0 \rightarrow \frac{(x-1)^2 - x^2}{x(x-1)} \geq 0 \rightarrow \frac{-2x+1}{x(x-1)} \geq 0$$

$$\frac{1}{x} - \frac{1}{x-1} \geq 0 \rightarrow D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$$

- 1

$$f(\star) = -2 \rightarrow \star^2 - 2\star = -2 \rightarrow (\star - 1)^2 = 0 \rightarrow \star = 1 \checkmark$$

$$g(a) = 2 \rightarrow \sqrt{a^2 - 1} = 2 \rightarrow a^2 - 1 = 4 \rightarrow a^2 = 5 \rightarrow a = \pm\sqrt{5}$$

$$x^2 = 2 \rightarrow x = 1 \rightarrow f(2) = 1^2 - 2 + 4 = 3$$

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$$x^2 = -5 \rightarrow x = -4 \rightarrow f(-5) = (-4)^2 - 2(-4) + 4 = 16 + 8 + 4 = 28$$

$$f(2) \times f(-5) = 3 \times 28 = 84$$