

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \geq 0 \Rightarrow \frac{x-1}{x} \geq \frac{x}{x-1} \rightarrow (x-1)^2 \geq x^2 \rightarrow x^2 - 2x + 1 \geq x^2$ $-2x + 1 \geq 0 \rightarrow x \leq \frac{1}{2}$ $f(x) = \frac{\frac{1}{x+1} - \frac{x}{x-1}}{\frac{x}{x-1} + \frac{1}{x+1}} \left\{ x \neq \left\{ 1, 0, \frac{1}{2}, -1 \right\} \right.$ $\left. D_f = \mathbb{R} - \left\{ -\frac{1}{2}, -1, 0, 1 \right\} \right\} \frac{x^2 + 1}{x-1} > 0 \rightarrow \frac{x^2 + 1}{(x-1)(x+2)} \rightarrow \begin{cases} x+2 \neq 0 \\ x \neq \frac{1}{2} \end{cases}$	1
$f(x) = \sqrt{\left(\left(\frac{1}{x}\right)^4 - 9\right)(x^2 - 4^x)} \geq 0 \Rightarrow x^2 - 4^x \geq 0 \rightarrow x^2 \geq 2^{2x} \rightarrow x = \frac{0}{2}$ $\left\{ \left(\frac{1}{x}\right)^4 - 9 \geq 0 \rightarrow \frac{1}{x^4} \geq 9 \rightarrow x = \frac{1}{3} + \frac{1}{3} - \frac{1}{3} \right\} (-\infty, -\frac{1}{3}] \cup \left[\frac{1}{3}, +\infty\right)$ $\sqrt{x-1} + \sqrt{y+1} = 3 \rightarrow \sqrt{y+1} = 3 - \sqrt{x-1} \rightarrow \sqrt{y+1} \geq 0 \rightarrow \sqrt{x-1} \leq 3$ $\downarrow$ $[x \geq 1]$ $\sqrt{y+1} = 3 - \sqrt{x-1}$ نامتقی $\sqrt{x-1} = 3 - \sqrt{y+1}$ نامتقی $\begin{cases} x-1 \leq 11 \\ x \leq 12 \end{cases} [1, 12]$ ✓	2
$f(x) = \frac{\log_f(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1} \left\{ \begin{array}{l} III \Rightarrow x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) > 0 \\ x > 2, x < -1 \end{array} \right.$ $I = \sqrt{x^2 - 1} \geq 0 \rightarrow x^2 \geq 1 \rightarrow x \geq 1, x \leq -1$ $D_f = (-\infty, -1) \cup (2, +\infty)$ $II = \sqrt{x^2 - 1} + 1 \neq 0 \rightarrow \sqrt{x^2 - 1} \neq -1$ بارنگال نیست است ✓	3
$\sqrt{\frac{1}{x+1} - x^2} - 2 \geq 0 \rightarrow \frac{1}{x+1} - x^2 \geq 0 \rightarrow -x^2 - 1 \geq 0 \rightarrow x^2 \leq -\frac{1}{x}$ $\Rightarrow -\left(x^2 - \frac{x}{x} - 2\right) \rightarrow \left(x - \frac{1}{x}\right)\left(x + \frac{2}{x}\right) \rightarrow \left[-2, \frac{2}{x}\right] \int_{-\frac{1}{x}}^{\frac{2}{x}} = 1$	4
$f(x) = \begin{cases} x^2 & x \geq 1 \\ 2x + 2 & x < 1 \end{cases} \quad g(x) = \begin{cases} x^2 - 2 - x \Rightarrow 2x^2 & x \geq 1 \\ x < 1 \Rightarrow 2x^2 - x \Rightarrow x + 2 & x \geq -2 \end{cases} \quad D_g = [-2, +\infty)$ $g(x) = \sqrt{f(x)} - x$	5

$$f\left(\sin \frac{\pi}{r} + \cos r\pi\right) = \underbrace{\left(2\left(\frac{\sqrt{r}}{r}\right) + 2(1)\right)}_{\sqrt{r}+2} \underbrace{\left(\frac{\sqrt{r}}{r} + 1 - 2\right)}_{\frac{\sqrt{r}}{r}-1} \Rightarrow \frac{r}{r} + \sqrt{r} - \sqrt{r} - 2$$

$$\sin \frac{\pi}{r} = \frac{\sqrt{r}}{r}$$

$$\cos r\pi = 1$$

$-\frac{1}{r}$

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$$f(a) = \frac{a+1}{a-1} \rightarrow f(f(a)) \rightarrow f\left(\frac{a+1}{a-1}\right) \rightarrow \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} \Rightarrow \frac{a+1+a-1}{a+1-a-1} = \frac{2a}{a-1}$$

$$\frac{f(a)}{a} = a \rightarrow \checkmark \quad \left. \vphantom{\frac{f(a)}{a} = a} \right\} R = \{1\}$$

$a \neq 1$

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$$g(a) = \sqrt{a^r - 1}$$

$$f(\sqrt{a^r - 1}) = (\sqrt{a^r - 1})^r - f\sqrt{a^r - 1} = -f \rightarrow a^r - 1 - f\sqrt{a^r - 1} + f = 0$$

$$(\sqrt{a^r - 1} - 2)^r = 0 \rightarrow \sqrt{a^r - 1} = 2 \rightarrow a^r - 1 = 4 \rightarrow a^r = 5 \rightarrow \boxed{a = \sqrt[r]{5}}$$

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$$\left. \begin{aligned} f(1+r) &= 1 - 0 + r \rightarrow r \\ f(-r+r) &= r^2 - r + r \rightarrow r^2 \end{aligned} \right\} f(r) \cdot f(-r) = r^2 \cdot r \rightarrow r^3$$

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$$f(\tan \alpha + \cot \alpha) = \tan \alpha \rightarrow \tan \alpha \Rightarrow \tan \frac{\pi}{4} = \textcircled{1}$$

$$r_1 t + 0 + - r_2 0 = 0$$

$$11t + 10t - 4 = 0 \Rightarrow (11t-4)(10t+4)$$

$$(11t-4)(10t+4) \rightarrow t = \frac{4}{11}, t = -\frac{4}{10}$$

$$\left. \vphantom{t = \frac{4}{11}} \right\} \cos \alpha = \frac{1}{r} \rightarrow \frac{r}{2}, \frac{\pi}{r}$$

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