

الف) $\frac{x-1}{x} - \frac{x}{x-1} \gg 0 \Rightarrow \frac{x^2-1-x^2}{x^2-x} \gg 0 \Rightarrow \frac{1-x}{x^2-x} \gg 0 \Rightarrow$

$D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب) $x \neq -2, -1, 0, 1$ $\frac{3}{x-1} + \frac{1}{x+2} \neq 0 \Rightarrow \frac{4x+5}{x^2+x-2} \neq 0 \Rightarrow 4x+5 \neq 0 \Rightarrow x \neq -\frac{5}{4}$

$D_f = \mathbb{R} - \{-2, -\frac{5}{4}, -1, 0, 1\}$

الف) $((\frac{1}{x})^x - 9)(22 - 4x) \gg 0 \Rightarrow$

$D_f = (-\infty, -2] \cup [\frac{5}{4}, +\infty)$

ب) $x-1 \gg 0 \Rightarrow x \gg 1$ $x-1 \ll 11 \Rightarrow x \ll 12 \Rightarrow D_f = [1, 12]$

$\sqrt{x^2-1} \neq -1$ ✓ همیشه بزرگتر

$x^2-x-2 > 0 \Rightarrow (x-2)(x+1) > 0$

$D_f = (-\infty, -1) \cup (2, +\infty)$

$\sqrt{-(x^2-ax-2)} \rightarrow x^2-ax-2 \ll 0 \rightarrow$ معین شلی:

$x = -1 \rightarrow 1 + a - 2 = 0 \Rightarrow a = 1 \Rightarrow a = -\frac{1}{2}$

$x^2 + \frac{x}{2} - 2 = 0 \rightarrow$

$x = -\frac{1}{2}$ $x = \frac{3}{2} \rightarrow b$

$a + b = \frac{3}{2} - \frac{1}{2} = 1$

$g(x) \rightarrow \sqrt{2x-1} \quad x \gg 1 \quad \checkmark$

$\sqrt{x+3} \quad x < 1 \quad x+3 \gg 0 \Rightarrow x \gg -3$

$D_f = [-3, 1) \cup [1, +\infty) = [-3, +\infty)$

$$f\left(\sin \frac{\pi}{r} + \cos rx\right) = r\left(1 + \frac{\sqrt{r}}{r}\right)^r - r\left(1 + \frac{\sqrt{r}}{r}\right) = r + \frac{r}{r} + r\sqrt{r} - r - r\sqrt{r} =$$

$$\sin \frac{\pi}{r} = \frac{\sqrt{r}}{r} \quad \cos 0 = 1 \quad \frac{r}{r} - r = -\frac{1}{r} \quad \boxed{-0/0}$$

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$$f(a) = \frac{a+1}{a-1} \quad f(f(a)) = a \Rightarrow \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = a \Rightarrow \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = a \Rightarrow a = a$$

$$a-1 \neq 0 \Rightarrow a \neq 1 \quad a \in \mathbb{R} \setminus \{1\}$$

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$$g(a) = \sqrt{a^2 - 1} \quad a^2 \geq 1 \Rightarrow a \geq 1, a \leq -1$$

$$f(g(a)) = (\sqrt{a^2 - 1})^r - r\sqrt{a^2 - 1} = -r \Rightarrow (\sqrt{a^2 - 1})^r - r\sqrt{a^2 - 1} + r = 0 \Rightarrow$$

$$(\sqrt{a^2 - 1} - r)^r = 0 \Rightarrow \sqrt{a^2 - 1} = r \Rightarrow a^2 = 0 \Rightarrow \boxed{a = \pm \sqrt{0}}$$

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$$f(1) = 1 - 0 + 1 = 2 \quad f(-1) = 1 + 0 + 1 = 2$$

$$f(1) \times f(-1) = 2 \times 2 = \boxed{4}$$

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$$r \cos^r x + 0 \cos x = r/0 \Rightarrow \cos x = \frac{1}{r} \rightarrow \sin x = \pm \frac{\sqrt{r}}{r}$$

$$\tan^r x = \frac{\left(\pm \frac{\sqrt{r}}{r}\right)^r}{\left(\frac{1}{r}\right)^r} = \frac{\frac{r}{r^r}}{\frac{1}{r^r}} = \boxed{r}$$

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