

الف) $\frac{x-1}{x} - \frac{x}{x-1} \geq 0 \rightarrow \frac{x^2 - 2x + 1 - x^2}{x(x-1)} \geq 0$

$\frac{-2x + 1}{x(x-1)} \geq 0$

پرس | + 0 - 1/2 + 1 -

$\rightarrow D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب) $\frac{x}{x-1} + \frac{1}{x+2} = 0 \rightarrow \frac{3x^2 + 5x - 2}{(x-1)(x+2)} = 0 \rightarrow x \neq -\frac{2}{3}$

$\rightarrow D_g = \mathbb{R} - \{0, -1, 1, -2, -\frac{2}{3}\}$

الف) $(x^2 - 9)(x^2 - 2x) \geq 0$

پرس | -2 2,5

$D_h = (-\infty, -2] \cup [2, 5, +\infty)$

ب) $\sqrt{x+1} = 3 - \sqrt{x-1}$

$\rightarrow \sqrt{x-1} \leq 3 \rightarrow 0 \leq x-1 \leq 9 \rightarrow 1 \leq x \leq 10 \rightarrow D_w = [1, 10]$

الف) $\frac{x^2 - x - 2}{\sqrt{x^2 - 1} + 1} \rightarrow x^2 - x - 2 > 0 \rightarrow x > 2, x < -1$

$x^2 - 1 \geq 0 \rightarrow x \geq 1, x \leq -1$

$\sqrt{x^2 - 1} + 1 \neq 0$ (همیشه مثبت)

$\rightarrow D_z = (-\infty, -1] \cup (2, +\infty)$

ب) $x = -\frac{1}{a} \rightarrow a = -\frac{1}{x} \rightarrow -x^2 - \frac{x}{x} + 3 = 0$

$x = \frac{\frac{1}{x} \pm \sqrt{\frac{1}{x^2} + 12}}{-2} = x = \frac{\frac{1}{x} \pm \frac{\sqrt{12x^2 + 1}}{x}}{-2} \rightarrow b = \frac{x}{x} \rightarrow a+b=1$

الف) $f(x) = x \geq 0 \rightarrow$

پرس | -2 1

$\rightarrow D_d = [-2, +\infty)$

$3x - 2 = x \rightarrow x = 1$

$2x + 3 = x \rightarrow x = -3$

$$f(\sin \frac{\pi}{2} + \cos \pi) = f(\sin 40^\circ + \cos 40^\circ)$$

$$f\left(\frac{\sqrt{x}+1}{x}\right) = x((x-1)^x - 1) = -\frac{1}{x}$$

$$f\left(\frac{a+1}{a-1}\right) = a \rightarrow \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = a \rightarrow \frac{\frac{2a}{a-1}}{\frac{2}{a-1}} = a = a$$

$$a-1 \neq 0$$

$$\frac{a+1}{a-1} - 1 \neq 0 \text{ (true)} \rightarrow \text{domain} = \{a \mid a \in \mathbb{R}, a \neq 1\}$$

$$|a^x - 1| < \varepsilon \rightarrow a^x - 1 = \varepsilon \rightarrow a^x = 1 + \varepsilon$$

$$a^x - 1 = -\varepsilon \rightarrow a^x = 1 - \varepsilon \rightarrow g(a) = x$$

$$a^x - 1 = \varepsilon \rightarrow a = \pm \sqrt[x]{1 + \varepsilon}$$

$$f(\varepsilon) = x, f(-\varepsilon) = y \rightarrow f(\varepsilon) \times f(-\varepsilon) = 1 \pm \varepsilon$$

$$\begin{cases} x+y = -x \\ x = -x \\ x^2 - 2x + 1 = 1 \end{cases} \Rightarrow x = 1$$

$$\varepsilon \cos x + \sin x - 1 = 0 \rightarrow \cos x + \sin x - 1 = 0$$

$$(\cos x - 1)(\cos x + 1) = 0 \quad \cos x = \frac{1}{2} \rightarrow -1 \leq \cos x \leq 1 \rightarrow \cos x = \frac{1}{2}$$

$$\rightarrow x = \frac{\pi}{3} \rightarrow \tan \frac{\pi}{3} = \sqrt{3}$$