

$$f(x) = x^2 - 5x \quad x \left(\frac{x+\sqrt{x}}{x} \right)^2 - 5 \left(\frac{x+\sqrt{x}}{x} \right) = \frac{x+\sqrt{x}}{x} - 5 - \sqrt{x}$$

$$x = \sin \frac{\pi}{x} + \cos x \pi = \frac{x+\sqrt{x}}{x}$$

$\downarrow \quad \downarrow$
 $\frac{\sqrt{x}}{x} \quad 1$

$$\frac{x+\sqrt{x}-x-5\sqrt{x}}{x} = \left(\frac{-1}{x} \right)$$

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$$f(x) = \frac{x+1}{x-1}$$

$$a \neq 1$$

$$f(f(a)) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{\frac{2a}{a-1}}{\frac{2}{a-1}} = a$$

$$\frac{a+1}{a-1} \neq 1$$

$$a+1 \neq a-1$$

$$1 \neq -1 \checkmark$$

$$R - \{1\}$$

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$$f(x) = x^2 - 5x \quad g(x) = \sqrt{x^2 - 1}$$

$$f(g(a)) = -5$$

$$a^2 - 5a = -5$$

$$a^2 - 5a + 5 = 0$$

$$(a-5)^2 = 0 \Rightarrow a=5 = g(a)$$

$$\sqrt{a^2 - 1} = 5 \Rightarrow a^2 - 1 = 25$$

$$a^2 = 26$$

$$a = \pm \sqrt{26}$$

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$$f(x+y) = x^2 - 2x + 5$$

$$x+y=5 \quad x=1 \rightarrow f(5) = 1^2 - 2(1) + 5 = 4$$

$$x+y=-5 \quad x=-9 \rightarrow f(-5) = (-9)^2 - 2(-9) + 5 = 88$$

$$f(5) \times f(-5) = 4 \times 88 = 352$$

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$$f(\cos x + a \cos x) = \tan^2 x$$

$$\cos^2 x + a \cos x = \tan^2 x$$

$$\cos^2 x + a \cos x - \tan^2 x = 0$$

$$(\cos^2 x + a \cos x - 1)(\cos^2 x - 1) = 0$$

$$\cos^2 x = 1$$

$$f\left(\frac{\pi}{4}\right) = \tan^2 \frac{\pi}{4} = (\sqrt{3})^2 = 3$$

$$\cos^2 x = -3, a \cos x = -1, \forall a \in \mathbb{R}$$

$$\cos^2 x = 1 \quad \cos x = \frac{1}{x} \quad x = \frac{\pi}{x}$$

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