

$$d) f(x) = \sqrt{\frac{x-1}{x} \cdot \frac{x}{x-1}} \quad \frac{x-1}{x} - \frac{x}{x-1} \geq 0 \quad \frac{x^2+1-x^2-x^2}{x(x-1)} \geq 0 \quad \frac{1-x^2}{x(x-1)} \geq 0 \quad \frac{1}{+\frac{1}{x} - \frac{1}{x} + \frac{1}{x} -}$$

$$\therefore f(x) = \frac{\frac{1}{x+1} - \frac{1}{x}}{\frac{1}{x-1} + \frac{1}{x+1}} \rightarrow \frac{x - x^2 - 1}{x(x+1)} \cdot \frac{(x-1)(x+1)}{(x-1)(x+1)} = \frac{-x^2 - x + 1}{(x-1)(x+1)}$$

$$\begin{aligned} \text{ii) } f(x) &= \sqrt{\left(\frac{1}{x}\right)^2 - 9} (12 - x^2) \quad \left(\frac{1}{x^2} - 9\right)(12 - x^2) \geq 0 \quad \frac{-2}{+4 - 4 +} \\ \frac{1}{x^2} &= 9 \quad x = -2 \quad x = \frac{0}{2} \\ \text{iii) } \sqrt{x-1} + \sqrt{y+1} &= 2 \\ x-1 &\geq 0 \quad y+1 \geq 0 \\ x &\geq 1 \end{aligned}$$

$$f(x) = \frac{\log_e(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$$

$x^2 - x - 2 > 0$ $(x-2)(x+1) > 0$
 $x^2 - 1 \geq 0$ $\sqrt{x^2 - 1} \neq -1$ (موازه برقراره)
 $(x+1)(x-1) \geq 0$
 $\frac{-1}{+} \frac{1}{-} \frac{1}{+}$

$(-\infty, -1) \cup (2, +\infty)$

$$f(x) = \sqrt{3+ax-x^2} \quad x=-2 \rightarrow \sqrt{3-2a-4} = 0 \quad \begin{array}{l} 2a = -1 \\ a = -\frac{1}{2} \end{array}$$

$$f(x) = \sqrt{3-\frac{1}{2}x-x^2} \rightarrow b-2 = 5_{\text{max}} = -\frac{1}{2}$$

$$b = \frac{5}{2} \quad \boxed{a+b = \frac{5}{2} - \frac{1}{2} = 1}$$

$$f(x) = \begin{cases} 4x-2 & x \geq 1 \\ 4x+3 & x < 1 \end{cases}$$

$$g(x) = \begin{cases} \sqrt{4x-2} & x \geq 1 \\ \sqrt{4x+3} & x < 1 \end{cases}$$

$$4x-2 \geq 0$$

$$x \geq \frac{1}{2}$$

$$4x+3 \geq 0$$

$$x \geq -\frac{3}{4}$$

$$g(x) = \sqrt{f(x)-x}$$

$$[-\frac{3}{4}, 1) \cup [1, +\infty) \rightarrow [-\frac{3}{4}, +\infty)$$

$$f(x) = x^2 - \varepsilon x \quad x \left(\frac{1+\sqrt{3}}{2} \right)^2 - \varepsilon \left(\frac{1+\sqrt{3}}{2} \right) = \frac{1+\varepsilon\sqrt{3}}{2} - \varepsilon - \frac{\varepsilon\sqrt{3}}{2}$$

$$x = \sin \frac{\pi}{3} + \cos 2\pi = \frac{1+\sqrt{3}}{2}$$

$\downarrow$ $\frac{\sqrt{3}}{2}$ $\downarrow$ 1

$$\frac{1+\varepsilon\sqrt{3}-1-\varepsilon\sqrt{3}}{2} = \left(\frac{-1}{2} \right)$$

6

$$f(x) = \frac{x+1}{x-1}$$

$$a \neq 1$$

$$f(f(a)) = \frac{\frac{a+1}{a-1}+1}{\frac{a+1}{a-1}-1} = \frac{\frac{2a}{a-1}}{\frac{2}{a-1}} = a$$

$$\frac{a+1}{a-1} \neq 1$$

$$a+1 \neq a-1$$

$$1 \neq -1 \checkmark$$

$$\boxed{\mathbb{R} - \{1\}}$$

7

$$f(x) = x^2 - \varepsilon x \quad g(x) = \sqrt{x^2 - 1}$$

$$f(g(a)) = -\varepsilon$$

$$a^2 - \varepsilon a = -\varepsilon$$

$$a^2 - \varepsilon a + \varepsilon = 0$$

$$(a-\varepsilon)^2 = 0 \Rightarrow a = \varepsilon = g(a)$$

$$\sqrt{a^2 - 1} = \varepsilon \Rightarrow a^2 - 1 = \varepsilon^2$$

$$a^2 = \varepsilon^2$$

$$\boxed{a = \pm \sqrt{\varepsilon^2}}$$

8

$$f(x+y) = x^2 - \omega x + \varepsilon$$

$$x+y = \varepsilon \quad x = 1 \rightarrow f(\varepsilon) = 1^2 - \omega(1) + \varepsilon = \varepsilon$$

$$x+y = -\varepsilon \quad x = -\varepsilon \rightarrow f(-\varepsilon) = (-\varepsilon)^2 - \omega(-\varepsilon) + \varepsilon = \varepsilon^2$$

$$f(\varepsilon) \times f(-\varepsilon) = \varepsilon^2 \times \varepsilon^2 = \boxed{1 \varepsilon^2}$$

9

$$f(\varepsilon \cos x + \omega \cos x) = \tan^2 x$$

$$f(\sqrt{3}, \omega) = \tan^2 \frac{\pi}{3} = (\sqrt{3})^2 = \boxed{3}$$

$$\varepsilon \cos^2 x + \omega \cos x = \sqrt{3}, \omega$$

$$\varepsilon \cos^2 x + \omega \cos x - \sqrt{3}, \omega = 0$$

$$(\varepsilon \cos^2 x + \sqrt{3}, \omega) (\varepsilon \cos^2 x - 1) = 0 \rightarrow$$

$$\varepsilon \cos^2 x = -\sqrt{3}, \omega \quad \cos^2 x = -1, \forall \omega \quad \text{impossible}$$

$$\varepsilon \cos^2 x = 1 \quad \cos^2 x = \frac{1}{\varepsilon} \quad x = \frac{\pi}{3}$$

10