

$$a) f(x) = \sqrt{\frac{x-1}{x}} = \frac{\sqrt{x-1}}{\sqrt{x}} \Rightarrow \sqrt{\frac{(x-1)^2 - x^2}{x^2 - x}} \Rightarrow \sqrt{\frac{-2x+1}{x^2-x}} \Rightarrow \frac{0}{\frac{1}{0} - \frac{1}{0} + \frac{1}{0}}$$

$$\Rightarrow D_f = (-\infty, 0) \cup (0, 1)$$

$$b) f(x) = \frac{\frac{1}{x^2} - \frac{1}{x}}{\frac{x}{x^2} + \frac{1}{x^2}} \quad \left. \begin{array}{l} \frac{1}{x^2} + \frac{1}{x^2} \Rightarrow x = \frac{0}{2} \\ b = (0, -1) \end{array} \right\} \Rightarrow D_f = \mathbb{R} - \left\{ -1, \frac{0}{2} = -1, 0, 1 \right\}$$

$$c) f(x) = \sqrt{\frac{1}{x^2} + 1} (x^2 - 1) \Rightarrow x = -1 \vee 0 \Rightarrow \frac{-1}{1} - \frac{1/0}{0} \Rightarrow D_f = (-\infty, -1] \cup (0, +\infty)$$

$$d) \sqrt{x^2 + 1} \sqrt{y+1} = x \Rightarrow (\sqrt{y+1})^2 = (\sqrt{x^2 + 1} - x)^2 \Rightarrow y+1 = (x^2 + 1 - 2x + x^2) \Rightarrow y = (x^2 - 2x) - 1 \Rightarrow y = x^2 - 2x - 1 \Rightarrow x \in \mathbb{R}$$

$$\Rightarrow \sqrt{x^2 + 1} \geq x \Rightarrow x \geq -1 \Rightarrow D_f = [-1, +\infty)$$

$$e) f(x) = \frac{1}{x^2 + 1} \quad \begin{array}{l} I \rightarrow x > 1 \\ II \rightarrow x < -1 \end{array} \quad \begin{array}{l} III \rightarrow x^2 - x^2 \geq 0 \Rightarrow \frac{0}{1} - \frac{0}{1} \\ IV \rightarrow x^2 - 1 \geq 0 \Rightarrow x \geq 1 \vee x \leq -1 \end{array}$$

$$f(x) = \frac{1}{x^2 + 1} \Rightarrow D_f = \mathbb{R} \quad \begin{array}{l} \frac{1}{x^2 + 1} = \frac{1}{x^2 + 1} \\ \frac{1}{x^2 + 1} = \frac{1}{x^2 + 1} \end{array}$$

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$$f(x) = x^r - rx \quad f\left(\sin \frac{\pi}{r} + i \cos \frac{\pi}{r}\right) = f\left(\sqrt{\frac{r}{r}} + i\right)$$

$$\Rightarrow r\left(\sqrt{\frac{r}{r}} + i\right)^r - r\left(\sqrt{\frac{r}{r}} + i\right) = r\left(1 + \frac{r}{r} + \sqrt{r}\right) \cdot r - r\sqrt{r} = r + \frac{r}{r} + r\sqrt{r} - r - r\sqrt{r} = -\frac{1}{r}$$

$$f(f(a)) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{\frac{2a}{a-1}}{\frac{2}{a-1}} = a \quad f(f(a)) \in \mathbb{Z} \Rightarrow a \in \mathbb{Z} - \{1\}$$

$$a \neq 1 \leftarrow \frac{a+1}{a-1} = 1 \Rightarrow a+1 = a-1 \rightarrow \emptyset \cup \emptyset$$

$$f(x) = x^r - rx$$

$$f(g(a)) = -r$$

$$\Rightarrow x^r - rx + r = 0 \Rightarrow x = r \quad \rightarrow g(a) \cup \emptyset$$

$$g(x) = \sqrt{x^r - 1}$$

$$g(x) = r$$

$$\Rightarrow \sqrt{x^r - 1} = r \Rightarrow x^r - 1 = r^2 \Rightarrow x^r = r^2 + 1 \Rightarrow x = \pm \sqrt[r]{r^2 + 1}$$

$$\Rightarrow a = \pm \sqrt[r]{r^2 + 1}$$

$$f(x+r) = x^r - rx + r$$

$$f(r) \times f(-r) = ? \quad \forall r \times r = 1 \times r$$

$$f(r) = 1 - r + r = r$$

$$f(-r) = r^r + r^0 + r = \sqrt{r}$$

$$f(\cos x + i \sin x) = \cos^r x$$

$$f(r, \omega)$$

$$\cos x = t$$

$$rt^r + \omega t - r/\omega = 0 \Rightarrow t = \frac{-\omega \pm \sqrt{\omega^2 + 4r\omega}}{2} = \frac{-1 \pm \sqrt{1+4r}}{2} = \frac{-1 \pm \sqrt{1+4r}}{2} \rightarrow \emptyset \cup \emptyset$$

$$\Rightarrow \cos x = \frac{1}{r} \Rightarrow x = \frac{\pi}{r} \in \left[0, \frac{\pi}{r}\right] \rightarrow \text{bijection}$$

$$\cos^r \frac{\pi}{r} = (\sqrt{r})^r = r$$