

۱۹، ۵۰ آفرین

۱) الف) $\frac{x^2 + 1 - 2x - x^2}{x^2 - x - 2} \rightarrow \frac{1 - 2x}{x(x-1)} \rightarrow \frac{\frac{1}{x} - 2}{1 - \frac{1}{x}} = \frac{1 - 2x}{x-1} - \frac{2}{1 - \frac{1}{x}} = \frac{1 - 2x}{x-1} - \frac{2x}{x-1} = \frac{1-4x}{x-1}$ (۰ و ۰) $\frac{1}{x} - 2$ (۰ و ۰)

ب) $\frac{x^2 - 2x - 3}{x^2 + x - 1} \rightarrow \frac{x(x-3)}{x(x+1)} = \frac{x-3}{x+1} = \frac{(x+2)^2 (x-1)}{x(x+1)(x+2)} \rightarrow \frac{x-1}{x+1}$
 $\frac{x^2 - 2x - 3}{x^2 + x - 1} = \frac{x(x-3)}{x(x+1)} = \frac{x-3}{x+1} = \frac{(x+2)^2 (x-1)}{x(x+1)(x+2)} \rightarrow \frac{x-1}{x+1}$
 $Df = R - d \left[\frac{1}{x} - 2 \right] - 1$

۲) الف) $\left(\frac{1}{x}\right)^x = 9 \rightarrow x = -2$
 $x^x = 9 \rightarrow x = \frac{1}{2}$

ب) $x-1 > 0 \rightarrow x > 1$
 $x+1 > 0 \rightarrow x > -1$
 $Df = d(x, 1) \parallel x > 1$
 $\sqrt{y+1} = 3 - \sqrt{x-1} \rightarrow \sqrt{x-1} \leq 3 \rightarrow x-1 \leq 9 \rightarrow x \leq 10$

۳) $x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) \rightarrow \frac{1}{x-2} - \frac{1}{x+1} > 0$
 $x^2 - 1 > 0 \rightarrow x^2 > 1 \rightarrow x > 1 \text{ or } x < -1$
 $Df = [1, 2] \cup [2, \infty)$

۴) $x^2 + ax - x^2 > 0 \rightarrow [2, 3] \rightarrow -2b = \frac{3}{-1} \Rightarrow -2b = -3 \Rightarrow b = \frac{3}{2}$
 $x+b=a \Rightarrow a=b-2 \rightarrow \frac{3}{2} - 2 = -\frac{1}{2}$
 $ab = -\frac{1}{2} + \frac{3}{2} = 1$

۵) $f(x) \cdot x > 0 \rightarrow \frac{2x-2-x}{2x+3-x} = \frac{x-2}{x+3} > 0 \rightarrow x > 2 \text{ or } x < -3$

۶) $\sin \frac{\pi}{4} \rightarrow \frac{\sqrt{2}}{2}$
 $\cos \frac{\pi}{4} \rightarrow \frac{\sqrt{2}}{2}$
 $f(x) = 2x^2 + x$
 $\left(1 + \frac{\sqrt{2}}{2}\right)^2 \Rightarrow \frac{1}{2} + \sqrt{2}$
 $2x^2 = \frac{1}{2} + x\sqrt{2}$
 $x = \frac{1}{2} + x\sqrt{2}$

$\forall) \quad a \neq f(a)$
 $\frac{a+1}{a-1} \neq 1, \quad a-1 = a+1$
 $f(f(a)) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = a$ R-2.1

$1) \quad f(0) \Rightarrow -x \rightarrow x^2 - 1 = -x \rightarrow (x-1)^2 = 0 \rightarrow x=1$
 $\sqrt{x^2-1} = x \rightarrow x^2 = 1 \rightarrow x = \pm 1$

$2) \quad f(x) \rightarrow x+y=1 \rightarrow x=1-y$
 $f(x) = 1 - \omega + y = 1$
 $x+y = -x \rightarrow y = -2x$
 $f(-x) = 1 - \omega + (-2x) = 1 - \omega - 2x$
 $f(x) \wedge f(-x) = 1 \times 1 = 1$

$3) \quad \cos^2 x + \cos x = \frac{1}{x} \rightarrow \cos x \rightarrow \frac{1}{x} \rightarrow \tan x \rightarrow \frac{1 - \frac{1}{x}}{\frac{1}{x}} = x$
 $\cos^2 x + \cos x = \frac{1}{x} \rightarrow (\cos x + 1)(\cos x - 1) \rightarrow \frac{1}{x}$
 $f\left(\frac{1}{x}\right) = x$

$4) \quad \sqrt{x-1} + \sqrt{y+1} = 2$
 $\sqrt{x-1} \geq 0 \rightarrow x \geq 1$
 $\sqrt{y+1} \geq 0 \rightarrow y \geq -1$
 $\sqrt{x-1} \leq 2 \rightarrow x \leq 5$

$I, II \rightarrow 1 \leq x \leq 5 \rightarrow D_f = [1, 5]$