

۱۹۵ آزمون

آراده قانون پرورد:

$$2f(a) = f(-2) + a \Rightarrow 2 \times 7 \times (a+1) = 4a - 4 \Rightarrow 14a + 14 = 4a - 4$$

$$10a = -18 \rightarrow a = \frac{-9}{5} \quad \text{سوال ۱}$$

$$f(2-\sqrt{3}) + f(2+\sqrt{3})$$

$$(2-\sqrt{3})(2+\sqrt{3}) = 1 \xrightarrow{\text{معادله مربع برابری}} \Rightarrow 2(2-\sqrt{3})f(2+\sqrt{3})$$

$$\left. \begin{aligned} f(2+\sqrt{3}) &= \\ \sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}} + 2 \end{aligned} \right\} \text{سوال ۲}$$

$$\Rightarrow 2(\sqrt{2}+2) = \frac{\sqrt{3}+1}{\sqrt{2}} + \frac{\sqrt{2}-1}{\sqrt{2}} = \frac{2\sqrt{3}}{\sqrt{2}} = \sqrt{6}+2$$

$$2f(x) - 2f(-x) = 4x^2 - x$$

$$\left. \begin{aligned} (2f(-x) - 2f(x) = 4x^2 + x) \times \frac{3}{2} \\ \Rightarrow f(x) = -4x^2 + \frac{1}{2}x \end{aligned} \right\} \text{سوال ۳}$$

$$\left. \begin{aligned} 2f(x) - 2f(-x) \\ = 4\Delta f(x) + 2f(-x) \end{aligned} \right\} \text{جمع}$$

$$\Rightarrow -2\Delta f(x) = 10x^2 + \frac{1}{2}x$$

$$\Rightarrow f(x) = -4x^2 + \frac{1}{2}x$$

$$x = -2 \rightarrow 2f(0) = 14 + 2m + 3m - 1$$

$$x = 0 \rightarrow (2f(0) = 3m - 1) \times 2$$

$$9m - 3 = 2m + 1 \Rightarrow m = 4$$

$$\Rightarrow 2f(0) = 3m - 1$$

$$\Rightarrow f(0) = \frac{2m-1}{2} = \frac{14,5}{2} \Rightarrow m = 6,25$$

$$f(x) + f\left(\frac{1}{x}\right) = -3x + \frac{2}{x} - 12$$

$$f(x) = ax + b, f\left(\frac{1}{x}\right) = \frac{a}{x} + b$$

$$\left. \begin{aligned} \frac{a}{x} + ax + 2b = -3x + \frac{2}{x} - 12 \Rightarrow a = -2, b = -4 \end{aligned} \right\} \text{سوال ۵}$$

$$\Rightarrow f(x) = -2x - 4 \Rightarrow f(-1) = 2 - 4 = -2$$

دقت کن!

$$x = -1 \rightarrow f(-1) + f(-1) = \frac{3(-1)^2 + 12 + 3}{-1} = -18 \rightarrow \boxed{f(-1) = -9} \quad \text{سوال ۵}$$

البته می توانیم عبارت را به فرم $f(x) = ax + b$ فرض کنیم

$$f(x) + f\left(\frac{1}{x}\right) = ax + b + \frac{a}{x} + b = \frac{ax^2 + bx + a + bx}{x} = \frac{3x^2 - 12x + 3}{x} \quad \begin{cases} a = 3 \\ b = -4 \end{cases}$$

$$f(x) = 3x - 4 \rightarrow \boxed{f(-1) = -9}$$

$$f(x) = (x-1)\sqrt{1-x}$$

$$g(x) = -\sqrt{(1-x)^3}$$

$$D_f = (-\infty, 1]$$

$$D_g = (-\infty, 1]$$

عددگذاری

$$f(0) = -1 \times \sqrt{1} = -1$$

$$g(0) = -\sqrt{(1-0)^3} = -\sqrt{1} = -1$$

سوال ۶

موردی که برابر است

$$f(x) = x\sqrt{x}$$

$$g(x) = \sqrt{x^3}$$

$$D_f = \mathbb{R}^+$$

$$D_g = \mathbb{R}^+$$

با عددگذاری عدد

$$f(2) = 2\sqrt{2} = \sqrt{2 \times 2}$$

$$g(2) = \sqrt{2^3} = \sqrt{2 \times 2}$$

سوال ۷

$$f(x) = [\sqrt{x} - \lfloor \sqrt{x} \rfloor]$$

$$g(x) = \sqrt{x - \lfloor x \rfloor}$$

$$D_f = \mathbb{R}^+$$

$$D_g = \mathbb{R}^+$$

عددگذاری

$$f(2) = [\sqrt{2} - \lfloor \sqrt{2} \rfloor] = 0$$

$$g(2) = \sqrt{2 - \lfloor 2 \rfloor} = 0$$

سوال ۸

الف) $\left(\frac{f+g}{f-g}\right)(1) = \frac{1+2}{2-1} = 3$

ب) $\left(\frac{f+g}{f-g}\right)(2) = \frac{3+3}{3-3} = \text{تن}$

سوال ۹

ج) $f(0)g(-1) = 0 \times g(-1) = 0$

الف) $f(2x-1) = \left\{ (1, 1), (0, 4), \left(\frac{1}{4}, 0\right), \left(-\frac{1}{3}, 2\right), \left(\frac{4}{3}, -1\right) \right\}$

سوال ۱۰

ب) $f(1 \times 1) \Rightarrow \left\{ (2, 1), (-2, 1), (0, 0), (3, -1), (-3, -1) \right\}$