

المطلوب

$$1) f(x) = \begin{cases} (a+1)(x+r) & ; x > 1 \\ ra + rx & ; x \leq 1 \end{cases}$$

$$rf(\omega) = f(-r) + a$$

$$f(\omega) = (a+1)r = ra + r$$

$$f(-r) = ra - r$$

$$(ra + r) = ra - r + a$$

$$10a = -18$$

$$a = -1/10$$

2)

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + r$$

$$f(x - \sqrt{x}) + f(x + \sqrt{x})$$

$$(x - \sqrt{x})(x + \sqrt{x}) = 1 \rightarrow x - \sqrt{x} = \frac{1}{x + \sqrt{x}}$$

$$f(x - \sqrt{x}) = \sqrt{x - \sqrt{x}} + \frac{1}{\sqrt{x - \sqrt{x}}} + r = \sqrt{x - \sqrt{x}} + \sqrt{x + \sqrt{x}} + r$$

$$f(x + \sqrt{x}) = \sqrt{x + \sqrt{x}} + \frac{1}{\sqrt{x + \sqrt{x}}} + r = \sqrt{x + \sqrt{x}} + \sqrt{x - \sqrt{x}} + r$$

$$f(x - \sqrt{x}) + f(x + \sqrt{x}) =$$

$$(\sqrt{x - \sqrt{x}} + \sqrt{x + \sqrt{x}})^2 = A$$

$$A^2 = 2 + 2\sqrt{1} = 4$$

$$A = \sqrt{4}$$

$$\rightarrow f(x - \sqrt{x}) + f(x + \sqrt{x}) = x + \sqrt{x} + \sqrt{x} + \sqrt{x} = 2x + 2\sqrt{x}$$

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$$rf(n) - rf(-n) = \xi n^r - n$$

$$-n \rightarrow rf(-n) - rf(n) = \xi n^r + n \xrightarrow{\times \frac{r}{r}} rf(-n) - \xi rf(n) = \xi n^r + 1/\omega n$$

exp $-rf(n) = \xi n^r + 1/\omega n$

$$f(n) = -\xi n^r + -1/r n$$

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$$(n+r)f(n) - rf(n+r) = \xi n^r - n n + r n - 1/r$$

$$f(0) = ?$$

$$n = a \rightarrow rf(a) = r n - 1 \xrightarrow{\times r} a n - r = rf(a)$$

$$n = -r \rightarrow rf(a) = 1/r + \omega n - 1 = a n + 1/\omega$$

$$a n - r = a n + 1/\omega$$

$$\xi n = 1/r$$

$$n = \xi / \omega$$

$$f(a) = \frac{r n - 1}{r} = \frac{rf(a)}{r} = 1/r \omega$$

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$$f(n) = a n + b$$

$$f\left(\frac{1}{n}\right) = \frac{a}{n} + b$$

$$f(n) + f\left(\frac{1}{n}\right) = a n + b + \frac{a}{n} + b = a n + \frac{a}{n} + 2b$$

$$= \frac{a n^r + r b n + a}{n} = \frac{r n^r - 1/r n + r}{n}$$

$$\begin{aligned} a &= r \\ b &= -1/r \end{aligned} \quad \left\{ \begin{aligned} f(n) &= r n - 1/r \\ f(1) &= -a \end{aligned} \right.$$

4)

$$f(x) = (x-1)\sqrt{1-x} \rightarrow D_f = (-\infty, 1]$$

$$g(x) = -\sqrt{(1-x)^3} = -\sqrt{(1-x)^2(1-x)} = -(1-x)\sqrt{1-x} \rightarrow D_g = (-\infty, 1]$$

$$x = -1 \rightarrow f(-1) = -2\sqrt{2}$$

$$g(-1) = -2\sqrt{2}$$

$$|1-x| \xrightarrow[\text{کسر از یزد}]{\text{به ازای}} = 1-x \rightarrow g(x) = (x-1)\sqrt{1-x}$$

دو تابع برابر ✓

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$$f(x) = x\sqrt{x} \rightarrow D_f = x \geq 0$$

$$g(x) = \sqrt{x^3} = |x|\sqrt{x} \rightarrow \text{در دامنه تابع همواره مثبت} \Rightarrow g(x) = x\sqrt{x}$$

$$\cancel{f(x) = x\sqrt{x}} \Rightarrow g(x) = x\sqrt{x}$$

دو تابع برابر ✓

۸)

$$f(x) = [\sqrt{x} - [\sqrt{x}]] \rightarrow D_f = x \geq 0$$

$$g(x) = \sqrt{x - [x]} \rightarrow D_g = x \geq 0$$

دو تابع به ازای هر مقداری برابر ✓

اگر x عدد صحیح باشد جواب صفری شود $x = 9 \rightarrow 3 - 3 = 0$

به ازای عدد اعشاری هم صفری شود $x = 4.5 \rightarrow [2.236] = 2 \rightarrow 2.236 - 2 = 0.236$

$$9) \quad g = \{(-1, 1), (1, 1), (2, 2), (3, 3), (4, 4), (5, 2), (\sqrt{5}, 1)\}$$

$$f = \{(1, 2), (2, 3), (3, 1), (4, 4)\}$$

$$1) \quad \left(\frac{f+g}{f-g} \right)(1) = \frac{f(1)+g(1)}{f(1)-g(1)} = 3$$

$$2) \quad \left(\frac{f+g}{f-g} \right)(2) = \frac{f(2)+g(2)}{f(2)-g(2)} = \text{تعريف نشده}$$

$$3) \quad f(a)g(-1) = 0 \times 1 = 0$$

10)

$$f(n) = \{(2, 1), (-1, 2), (4, 0), (-2, 2), (3, -1)\}$$

$$1) \quad g(n) = f(3n-1)$$

$$\{(1, 1), (0, 2), \left(\frac{1}{3}, 0\right), \left(-\frac{1}{3}, 2\right), \left(\frac{2}{3}, -1\right)\}$$

$$3n-1=2 \quad n=1 \quad 3n-1=1 \quad n=0 \quad 3n-1=-2 \quad n=-\frac{1}{3}$$

$$2) \quad h(n) = f(|n|)$$

$$\{(2, 1), (0, 0), (3, -1), (-2, 1), (-3, -1)\}$$