

$$f(x) = \begin{cases} (x+1)(x+2) & x > 1 \\ 3x + 2x & x \leq 1 \end{cases} \quad \left\{ \begin{array}{l} 2f(1) = f(-1) + 4 \Rightarrow 2(1+1)(1+2) = 3(-1) + 2(-1) \\ |f_{x+1}|^2 = f_{x-1} \rightarrow |1+2| = |-1| \rightarrow x = -1/2 \end{array} \right.$$

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$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2 \quad f(x-\sqrt{x}) + f(x+\sqrt{x}) \rightarrow \frac{1}{\sqrt{x-\sqrt{x}}} + \frac{\sqrt{x+\sqrt{x}}}{\sqrt{x+\sqrt{x}}} = \frac{\sqrt{x+\sqrt{x}}}{\sqrt{x-\sqrt{x}}} = \sqrt{x+\sqrt{x}}$$

$$\sqrt{x-\sqrt{x}} + \frac{1}{\sqrt{x-\sqrt{x}}} + 2 + \sqrt{x+\sqrt{x}} + \frac{1}{\sqrt{x+\sqrt{x}}} + 2 \Rightarrow \frac{1}{\sqrt{x-\sqrt{x}}} = \sqrt{x-\sqrt{x}}$$

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$$2(\sqrt{x-\sqrt{x}} + \sqrt{x+\sqrt{x}} + 2) \rightarrow (\sqrt{x-\sqrt{x}} + \sqrt{x+\sqrt{x}})^2 \rightarrow 4 + 2 \rightarrow 6 \rightarrow 2(\sqrt{6+2}) \rightarrow 2\sqrt{8} = 4\sqrt{2}$$

$$\left. \begin{array}{l} 2f(x) - 3f(-x) = f_{x^2} - 4 \rightarrow 2f(x) - 3f(-x) = 14x^2 - 2x \\ 3f(-x) - 2f(x) = f_{x^2} + 4 \rightarrow 3f(-x) - 2f(x) = 14x^2 + 2x \end{array} \right\} \xrightarrow{+} -5f(x) = 20x^2 + 4x$$

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$$f(x) = 4x^2 + \frac{4}{x}$$

$$(x+1)f(x) - 3xf(x+1) = f_{x^2} - mx + m-1$$

$$x=0 \rightarrow f(0) = 0 = m-1$$

$$x=1 \rightarrow 0 - 2f(0) = 14 + m - m-1 \rightarrow -2f(0) = 14 \rightarrow f(0) = -7$$

$$\left. \begin{array}{l} f(0) = 9m-1 \\ + f(0) = 10m+0m \end{array} \right\} \quad 9m-1 = 10m \rightarrow m = -1 \rightarrow m = \frac{1}{5} \rightarrow \frac{9}{5} = \frac{9}{5}$$

$$\rightarrow f(x) = \frac{3m-1}{5} = \frac{3(-1)-1}{5} = \frac{-4}{5} = -\frac{4}{5}$$

$$f(m) + f\left(\frac{1}{a}\right) = \frac{3a^2 - 14a + 3}{a} \rightarrow a^2 + b + \frac{a}{a} + b = 3a - 14 + \frac{3}{a} \rightarrow a^2 + \frac{a}{a} + b = 3a - 14 + \frac{3}{a}$$

$$\Rightarrow a = 3, b = -4$$

$$f(x) = 3x - 4 \rightarrow f(-1) = -3 - 4 = -7$$

$$f(x) = (x-1)\sqrt{1-x} \rightarrow D_f = 1-x \geq 0 \rightarrow x \leq 1 \checkmark$$

$$g(x) = -\sqrt{1-x} \rightarrow D_g = 1-x \geq 0 \rightarrow x \leq 1 \checkmark$$

$$q(x) = -(1-x)\sqrt{1-x} \rightarrow (x-1)\sqrt{1-x}$$

$$\left\{ D_f = D_g = [-\infty, 1] \right. \quad f=g$$

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$$f(x) = x\sqrt{x} \rightarrow D_f = [0, +\infty)$$

$$g(x) = \sqrt{x^3} \rightarrow D_g = [0, +\infty)$$

$$\left\{ D_f = D_g = [0, +\infty) \right. \quad \checkmark$$

$$q(x) = x\sqrt{x} \quad \checkmark \quad g$$

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$$f(x) = [\sqrt{x} - \lfloor \sqrt{x} \rfloor] \rightarrow \sqrt{x} \geq 0 \rightarrow x \geq 0 \rightarrow 0 \leq \sqrt{x} - \lfloor \sqrt{x} \rfloor < 1 \rightarrow f(x) = 0$$

$$g(x) = \sqrt{x-|x|} \rightarrow x-|x| \geq 0 \rightarrow x \geq |x| \left\{ \begin{array}{l} x \geq 0 \rightarrow x \geq x \rightarrow g(x) = 0 \\ x < 0 \rightarrow x \geq -x \rightarrow x \geq 0 \rightarrow g(x) = 0 \end{array} \right.$$

$$f=g \quad \checkmark$$

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$$f = \{(1,2), (2,3), (3,4), (4,5)\} \cdot g = \{(-1,1), (1,1), (2,3), (3,0), (0,2), (\sqrt{2},1)\}$$

$$\left(\frac{f+g}{f-g} \right) \rightarrow \frac{f(1)+g(1)}{f(1)-g(1)} \rightarrow \frac{2+1}{2-1} \rightarrow \text{دقت! } 3$$

$$\left(\frac{f+g}{f-g} \right)^2 \rightarrow \frac{f(2)+g(2)}{f(2)-g(2)} \rightarrow \frac{3+3}{3-3} \rightarrow \text{تعیین نشد}$$

$$f(-1)g(-1) = 0 \wedge 1 > 0$$

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$$f(x) = \{(2,1), (-1,2), (0,0), (-2,2), (5,-1)\}$$

$$g(x) = f(x^2-1) \rightarrow g(x) = \{(2,0), (-1,-4), (-2,-7), (3,1)\}$$

باید ۱-۳ را برابر D_f قرار دهی تا f و g همی جدید باشند

$$h(x) = f(|x|) \rightarrow h(x) = \{(2,1), (1,-4), (0,-7), (2,2), (3,3)\}$$

$$h(-1) = f(1) \times$$

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$$x=0 \quad 2f(0) = 3m-1 \rightarrow f(0) = \frac{3m-1}{2}$$

$$x=-2 \quad 4f(0) = 4(-2)^2 + 2m + 3m-1 \rightarrow f(0) = \frac{5m+15}{4}$$

$$\frac{5m+15}{4} = \frac{3m-1}{2} \rightarrow 1.5m + 3.75 = 1.5m - 0.5 \rightarrow 1.5m = 3.25 \rightarrow m = \frac{4}{3} \text{ و } 4.5$$

$$f(0) = \frac{3m-1}{2} \text{ و } \frac{15+5m}{4} \rightarrow f(0) = \frac{3(4.5)-1}{2} = \boxed{4.25 \text{ و } \frac{25}{2}}$$

۱۰- الف) یک اگرمسازیم و طبق آن n های را پیدا میکنیم نه f آن ها را میزنند.

$$3n-1=t \rightarrow n = \frac{t+1}{3} \quad f(3n-1) = \left\{ \left(\frac{t+1}{3}, 1 \right) \left(\frac{-t+1}{3}, 4 \right) \left(\frac{t+1}{3}, 5 \right) \left(\frac{-t+1}{3}, 2 \right) \left(\frac{t+1}{3}, -1 \right) \right\}$$

$$f(3n-1) = \left\{ (1, 1) (0, 4) \left(\frac{1}{3}, 5 \right) \left(\frac{-1}{3}, 2 \right) \left(\frac{4}{3}, -1 \right) \right\}$$

۱۰- ب) مثل زمانی که $f(1n)$ را رسم میزنیم باید n های منفی را حذف کنیم، n های باقی صافه را میزنیم

پس -2 و -1 را دامنه حذف میزنند و باقی n ها قرار میزنند

$$h(n) = \left\{ (\pm 2, 1) (0, 5) (\pm 3, -1) \right\} = \left\{ (-2, 1) (2, 1) (0, 5) (3, -1) (-3, -1) \right\}$$

و واقعاً f دیگر n های منفی را نمیزنند و آن ها را به فرم یک عدد مثبت میزنند