

۲. افزاین عالی نوشتی

$$f(x) = \begin{cases} (a+1)(x+2) & x > 1 \\ 3a + 2x & x \leq 1 \end{cases}$$

$$2f(5) = (a+1)(7) \times 2$$

$$f(-2) + a = 6a - 4$$

$$14a + 14 = 6a - 4$$

$$10a = -18$$

$$a = -1.8$$

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$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2 \quad f(2-\sqrt{3}) + f(2+\sqrt{3}) = \sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}} +$$

$$\frac{1}{\sqrt{2-\sqrt{3}}} + \frac{1}{\sqrt{2+\sqrt{3}}} + 4 = \sqrt{6} + \sqrt{6} + 4 = 2\sqrt{6} + 4$$

$$(\sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}})^2 = 2\sqrt{3} + 2 - \sqrt{3} + 2\sqrt{1} = 6$$

$$\left(\frac{1}{\sqrt{2-\sqrt{3}}} + \frac{1}{\sqrt{2+\sqrt{3}}}\right)^2 = \frac{1}{2-\sqrt{3}} + \frac{1}{2+\sqrt{3}} + 2\left(\frac{1}{\sqrt{(2-\sqrt{3})(2+\sqrt{3})}}\right) = 6$$

$$2f(x) - 3f(-x) = 5x^2 - 2x$$

$$(2f(-x) - 3f(x) = 5x^2 + 2x) \times \frac{3}{2} \rightarrow 3f(-x) - \frac{9}{2}f(x) = 5x^2 + \frac{3}{2}x$$

$$-\frac{5}{2}f(x) = 10x^2 + \frac{3}{2}x$$

$$f(x) = -2x^2 - \frac{1}{5}x$$

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$$(x+2)f(x) - 3xf(x+2) = 5x^2 - mx + 3m - 1$$

$$x=0 \rightarrow 2f(0) = 3m - 1 \quad f(0) = \frac{3}{2}m - \frac{1}{2} = \frac{3}{2}\left(\frac{9}{2}\right) - \frac{1}{2} = \frac{27}{2} - \frac{1}{2} = \frac{26}{2} = 13$$

$$x=-2 \rightarrow 4f(0) = 14 + 2m + 3m - 1 = 13 + 5m$$

$$9m - 3 = 13 + 5m$$

$$4m = 16 \quad m = 4$$

$$f(x) = ax + b$$

$$f\left(\frac{1}{x}\right) = \frac{a}{x} + b$$

$$ax + 2b + \frac{a}{x} = 2x - 12 + \frac{3}{x}$$

$$a = 2$$

$$2b = -12$$

$$b = -6$$

$$f(x) = 2x - 6$$

$$f(-1) = -8$$

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| $f(x) = (x-1)\sqrt{1-x} \quad x \leq 1 \quad D_f = (-\infty, 1]$<br>$g(x) = -\sqrt{(1-x)^3} \quad (1-x)^3 \geq 0 \quad 1-x \geq 0 \quad x \leq 1 \quad D_g = (-\infty, 1]$<br>برای همه اعداد نیز مقدار برابر دارند<br>$\boxed{f=g}$                                                                                                                                                                                                                                                                                                                | ۶  |
| $f(x) = x\sqrt{x} \quad x \geq 0 \quad D_f = [0, +\infty)$<br>$g(x) = \sqrt{x^3} \quad x^3 \geq 0 \quad x \geq 0 \quad D_g = [0, +\infty)$<br>$\hookrightarrow  x \sqrt{x} \xrightarrow{x \geq 0} x\sqrt{x}$<br>$\boxed{f=g}$                                                                                                                                                                                                                                                                                                                      | ۷  |
| $f(x) = [\sqrt{x} - \lfloor \sqrt{x} \rfloor] \quad x \geq 0 \quad D_f = [0, +\infty)$<br>$g(x) = \sqrt{x - \lfloor x \rfloor} \quad x - \lfloor x \rfloor \geq 0 \quad x \geq \lfloor x \rfloor \quad x \geq 0 \quad D_g = [0, +\infty)$<br>$g(x) = \sqrt{x - x} = 0$<br>$f(x) \rightarrow 0 \leq \sqrt{x} \cdot [\sqrt{x}] < 1 \Rightarrow [\sqrt{x} - \lfloor \sqrt{x} \rfloor] = 0$<br>$\boxed{f=g}$                                                                                                                                           | ۸  |
| الف) $\left(\frac{f+g}{f-g}\right)(1) = \frac{(f+g)(1)}{(f-g)(1)} = \frac{f(1)+g(1)}{f(1)-g(1)} = \frac{2+1}{2-1} = 3$<br>ب) $\left(\frac{f+g}{f-g}\right)(2) = \frac{(f+g)(2)}{(f-g)(2)} = \frac{2+3}{3-3} = \frac{5}{0} \rightarrow$ تعریف نشده<br>ج) $f(0)g(-1) = 0 \times 1 = 0$                                                                                                                                                                                                                                                               | ۹  |
| $g(x) = f(3x-1) \quad \begin{matrix} 3x-1=2 & 3x-1=-1 & 3x-1=0 & 3x-1=-2 \\ x=1 & x=0 & x=\frac{1}{3} & x=-\frac{1}{3} \end{matrix}$<br>$3x-1=3 \Rightarrow x=\frac{4}{3} \quad g(x) = \left\{ (1,1) \left(0, \frac{1}{3}\right) \left(\frac{1}{3}, 0\right) \left(-\frac{1}{3}, -2\right) \left(\frac{4}{3}, -1\right) \right\}$<br>$h(x) = f( x )$<br>$\begin{matrix}  x =2 &  x =-1 \text{ و } 1 &  x =0 &  x =-2 \text{ و } 2 &  x =3 & x=\pm 3 \\ x=\pm 2 & & x=0 & & \end{matrix}$<br>$\boxed{h(x) = \{(2,1) (-2,1) (0,0) (2,-1) (-2,-1)\}}$ | ۱۰ |