

$$f(x) = \begin{cases} (a+1)(x+2) & x > 1 \\ 2a + 2x & x \leq 1 \end{cases} \quad \begin{aligned} 2f(5) &= (a+1)(7) \times 2 \\ f(-2) + a &= 2a - 4 \end{aligned}$$

$$14a + 14 = 4a - 4$$

$$10a = -18$$

$$a = -1.8$$

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$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2 \quad f(2-\sqrt{3}) + f(2+\sqrt{3}) = \sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}} +$$

$$\frac{1}{\sqrt{2-\sqrt{3}}} + \frac{1}{\sqrt{2+\sqrt{3}}} + 4 = \sqrt{6} + \sqrt{6} + 4 = (4 + 2\sqrt{6})\sqrt{6}$$

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$$(\sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}})^2 = 2\sqrt{3} + 2 - \sqrt{3} + 2\sqrt{1} = 6 \quad \left(\frac{1}{\sqrt{2-\sqrt{3}}} + \frac{1}{\sqrt{2+\sqrt{3}}}\right)^2 = \frac{1}{2-\sqrt{3}} + \frac{1}{2+\sqrt{3}} + 2\left(\frac{1}{\sqrt{2-\sqrt{3}}}\right)\left(\frac{1}{\sqrt{2+\sqrt{3}}}\right) = 6$$

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$$\begin{aligned} 2f(x) - 3f(-x) &= 5x^2 - 2x \\ (2f(-x) - 3f(x) &= 5x^2 + 2x) \times \frac{2}{5} \rightarrow 3f(-x) - \frac{4}{5}f(x) = 5x^2 + \frac{4}{5}x \\ -\frac{1}{5}f(x) &= 10x^2 + \frac{4}{5}x \\ f(x) &= -5x^2 - \frac{4}{5}x \end{aligned}$$

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$$\begin{aligned} (x+2)f(x) - 3xf(x+2) &= 5x^2 - mx + 3m - 1 \\ x=0 \rightarrow 2f(0) &= 3m - 1 \quad f(0) = \frac{m}{2}m - \frac{1}{2} = \frac{m}{2}\left(\frac{m}{2}\right) - \frac{1}{2} = \frac{m^2}{4} - \frac{1}{2} \\ x=-2 \rightarrow 4f(0) &= 14 + 2m + 3m - 1 = 13 + 5m = \left(\frac{m^2}{4}\right) \\ 9m - 3 &= 13 + 5m \\ 4m &= 16 \quad m = 4 \end{aligned}$$

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$$\begin{aligned} f(x) &= ax + b \\ f\left(\frac{1}{x}\right) &= \frac{a}{x} + b \\ ax + 2b + \frac{a}{x} &= 2x - 12 + \frac{2}{x} \\ a &= 2 \\ 2b &= -12 \\ b &= -6 \\ f(x) &= 2x - 6 \\ f(-1) &= -8 \end{aligned}$$

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| $f(x) = (x-1)\sqrt{1-x} \quad x \leq 1 \quad D_f = (-\infty, 1]$<br>$g(x) = -\sqrt{(1-x)^3} \quad (1-x)^3 \geq 0 \quad 1-x \geq 0 \quad x \leq 1 \quad D_g = (-\infty, 1]$<br>برای همه اعداد نیز مقدار برابر دارند<br><div style="border: 1px solid black; padding: 5px; display: inline-block;"><math>f=g</math></div>                                                                                                                                                                                                                                                                                              | ۶  |
| $f(x) = x\sqrt{x} \quad x \geq 0 \quad D_f = [0, +\infty)$<br>$g(x) = \sqrt{x^3} \quad x^3 \geq 0 \quad x \geq 0 \quad D_g = [0, +\infty)$<br>$\hookrightarrow  x \sqrt{x} \xrightarrow{x \geq 0} x\sqrt{x}$<br><div style="border: 1px solid black; padding: 5px; display: inline-block;"><math>f=g</math></div>                                                                                                                                                                                                                                                                                                    | ۷  |
| $f(x) = [\sqrt{x} - \lfloor \sqrt{x} \rfloor] \quad x \geq 0 \quad D_f = [0, +\infty)$<br>$g(x) = \sqrt{x - \lfloor x \rfloor} \quad x - \lfloor x \rfloor \geq 0 \quad x \geq \lfloor x \rfloor \quad x \geq 0 \quad D_g = [0, +\infty)$<br>$g(x) = \sqrt{x - x} = 0$<br>$f(x) \rightarrow 0 \leq \sqrt{x} \cdot [\sqrt{x}] < 1 \Rightarrow [\sqrt{x} - \lfloor \sqrt{x} \rfloor] = 0$<br><div style="border: 1px solid black; padding: 5px; display: inline-block;"><math>f=g</math></div>                                                                                                                         | ۸  |
| الف) $\left(\frac{f+g}{f-g}\right)(1) = \frac{(f+g)(1)}{(f-g)(1)} = \frac{f(1)+g(1)}{f(1)-g(1)} = \frac{2+1}{2-1} = 3$<br>ب) $\left(\frac{f+g}{f-g}\right)(2) = \frac{(f+g)(2)}{(f-g)(2)} = \frac{2+3}{3-2} = \frac{5}{1} \rightarrow$ تعریف نشده<br>ج) $f(0)g(-1) = 0 \times 1 = 0$                                                                                                                                                                                                                                                                                                                                 | ۹  |
| $g(x) = f(3x-1) \quad \begin{matrix} 3x-1=2 & 3x-1=-1 & 3x-1=0 & 3x-1=-2 \\ x=1 & x=0 & x=\frac{1}{3} & x=-\frac{1}{3} \end{matrix}$<br>$3x-1=3 \Rightarrow x=\frac{4}{3} \quad g(x) = \left\{ (1,1), (0,2), \left(\frac{1}{3},0\right), \left(-\frac{1}{3},2\right), \left(\frac{4}{3},-1\right) \right\}$<br>$h(x) = f( x )$<br>$\begin{matrix}  x =2 &  x =-1 \text{ و } 1 &  x =0 &  x =-2 \text{ و } 2 &  x =3 & x=\pm 3 \\ x=\pm 2 & & x=0 & & \end{matrix}$<br><div style="border: 1px solid black; padding: 5px; display: inline-block;"><math>h(x) = \{(2,1), (-2,1), (0,0), (2,-1), (-2,-1)\}</math></div> | ۱۰ |