

نام و نام خانوادگی: شماره: کلاس: پاسخنامه تشریحی

$$\varphi(x) = \begin{cases} (a+1)(x+2) & x > 1 \\ 3a+2x & x \leq 1 \end{cases} \quad \begin{aligned} 2(a+1)(1) &= 3a-1+a \\ 1(a+1)+1 &= 1a-1 \Rightarrow a = -\frac{11}{10} = -\frac{9}{10} \end{aligned}$$

$$2\varphi(5) = \varphi(-2) + a$$

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$$\varphi(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2 \quad \varphi(2-\sqrt{3}) + \varphi(2+\sqrt{3}) = ? \quad \frac{1}{2+\sqrt{3}} = 2-\sqrt{3}, \quad 2+\sqrt{3} = \frac{1}{2-\sqrt{3}}$$

$$\left. \begin{aligned} \varphi(2+\sqrt{3}) &= \sqrt{2+\sqrt{3}} + \frac{1}{\sqrt{2+\sqrt{3}}} + 2 = \sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}} + 2 \\ \varphi(2-\sqrt{3}) &= \sqrt{2-\sqrt{3}} + \frac{1}{\sqrt{2-\sqrt{3}}} + 2 = \sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}} + 2 \end{aligned} \right\} \Rightarrow \varphi(2+\sqrt{3}) + \varphi(2-\sqrt{3}) = 2(\sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}}) + 4$$

$$A = \sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}} \Rightarrow \varphi(2+\sqrt{3}) + \varphi(2-\sqrt{3}) = 2\sqrt{2} + 4$$

$$A^2 = 2+\sqrt{3} + 2-\sqrt{3} + 2 \Rightarrow A = \sqrt{2}$$

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$$\begin{aligned} 2\varphi(1-x) - 3\varphi(1-x) &= 1-x^2-x \\ 2\varphi(1-x) - 3\varphi(x) &= 1-x^2+x \end{aligned} \rightarrow \begin{cases} 2\varphi(1-x) - 3\varphi(x) = 1-x^2-x \\ 3\varphi(1-x) - 9\varphi(x) = 7x^2+15x \end{cases}$$

$$-\frac{1}{7}\varphi(1-x) = 10x^2 + \frac{1}{7}x \Rightarrow \varphi(x) = -10x^2 - \frac{1}{7}x$$

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$$(x+2)\varphi(x) - 3x\varphi(x+2) = 1-x^2-mx+3m-1$$

$$\begin{aligned} x=-2 &\rightarrow 7\varphi(0) = 17+2m+3m-1 \Rightarrow 7\varphi(0) = 5m+16 \Rightarrow \varphi(0) = \frac{5m+16}{7} \\ x=0 &\rightarrow 2\varphi(0) = 3m-1 \end{aligned}$$

$$\rightarrow \begin{cases} 7\varphi(0) = 5m+16 \\ 2\varphi(0) = 3m-1 \end{cases} \rightarrow \begin{cases} 7\varphi(0) = 5m+16 \\ 7\varphi(0) = 9m-3 \end{cases}$$

$$\Rightarrow 5m+16 = 9m-3 \Rightarrow m = \frac{19}{4} = 4.75 \Rightarrow \varphi(0) = \frac{25}{4} = 6.25$$

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$$\varphi\left(\frac{1}{x}\right) + \varphi(x) = \frac{3x^2-12x+3}{x} \rightarrow ax+b + \frac{a}{x} + b = 3x-12 + \frac{3}{x} \Rightarrow a=3$$

$$\varphi(x) = ax+b \quad \varphi(x) = 3x-7 \Rightarrow \varphi(-1) = -9 \quad -2b=12 \Rightarrow b=-6$$

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$$\begin{cases} f(x) = (x-1)\sqrt{1-x} \rightarrow 1-x \geq 0 \Rightarrow x \leq 1 \text{ ب } D_f = (-\infty, 1] \\ g(x) = -\sqrt{(1-x)^3} \rightarrow (1-x)^3 \geq 0 \Rightarrow 1-x \geq 0 \Rightarrow x \leq 1 \text{ ب } D_g = (-\infty, 1] \end{cases}$$

اگر $x = -1 \Rightarrow (-1-1)\sqrt{1-(-1)} = -\sqrt{(1-(-1))^3} \rightarrow -2\sqrt{2} = -2\sqrt{2} \Rightarrow$

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$$\begin{cases} f(x) = x\sqrt{x} \rightarrow x \geq 0 \text{ ب } D_f = [0, +\infty) \\ g(x) = \sqrt{x^3} \rightarrow x^3 \geq 0 \Rightarrow x \geq 0 \text{ ب } D_g = [0, +\infty) \end{cases}$$

اگر $x = 2 \Rightarrow 2\sqrt{2} = \sqrt{8} \Rightarrow$

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$$\begin{cases} f(x) = [\sqrt{x} - \lfloor \sqrt{x} \rfloor] \rightarrow x \geq 0 \text{ ب } D_f = [0, +\infty) \cdot \sqrt{x} = t \quad f(x) = [t - \lfloor t \rfloor] \\ g(x) = \sqrt{x - |x|} \end{cases}$$

خطی که همیشه $0 \leq t - \lfloor t \rfloor < 1$ را نشان می‌دهد و این عبارت را می‌توان صفر نوشت

اگر $x - |x| \geq 0 \rightarrow |x| \leq x \Rightarrow x \geq 0 \text{ ب } D_g = [0, +\infty)$

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$$f = \{(1, 2), (2, 3), (3, 1), (0, 0)\} \quad g = \{(-1, 1), (1, 1), (2, 3), (3, 5), (0, 2), (\sqrt{5}, 1)\}$$

الف) $\left(\frac{f+g}{f-g}\right)(1) = \frac{f(1)+g(1)}{f(1)-g(1)} = \frac{2+1}{2-1} = 3$

ب) $\left(\frac{f+g}{f-g}\right)(2) = \frac{f(2)+g(2)}{f(2)-g(2)} = \frac{3+3}{3-3} = \frac{6}{0} \Rightarrow$ تعریف نشده است

ج) $f(0)g(-1) = 0 \times 1 = 0$

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$$f(x) = \{(2, 1), (-1, 4), (0, 5), (-2, 2), (3, -1)\}$$

الف) $g(x) = f(3x-1) \Rightarrow$ باید در اینجا $3x-1$ را به جای x در f قرار دهیم

ب) $h(x) = f(f(x)) \Rightarrow$ چون $f(x)$ باید اعضای نامنفی داشته باشد

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