

→ مسئله

1)

$$f(n) = \sqrt{n + \epsilon \sqrt{n - \epsilon}}$$

$$g(n) = \sqrt{n - \epsilon \sqrt{n - \epsilon}}$$

$$g(n) = \sqrt{n - \epsilon \sqrt{n - \epsilon}} = \sqrt{n - \epsilon \sqrt{n - \epsilon - \epsilon + \epsilon}}$$

$$= \sqrt{(\sqrt{n - \epsilon} - \epsilon)^2} = |\sqrt{n - \epsilon} - \epsilon|$$

با بساز
روش

$$f(n) = |\sqrt{n - \epsilon} + \epsilon|$$

$$g(n) + f(n) = |\sqrt{n - \epsilon} - \epsilon| + |\sqrt{n - \epsilon} + \epsilon|$$

برای ϵ محدود
عبارت مثبت

$$= 2\sqrt{n - \epsilon} = a + b\sqrt{n + c}$$

$$\left. \begin{array}{l} a = 0 \\ b = 2 \\ c = -\epsilon \end{array} \right\} \frac{a+b}{c} = \frac{2}{-\epsilon} = \left(-\frac{1}{\epsilon} \right)$$

2)

$$f(n) = \frac{n + \epsilon}{n^2 - \epsilon n + \epsilon} = \frac{n + \epsilon}{(n - \epsilon)(n - 1)}$$

$$D_f = \mathbb{R} - \{\epsilon, 1\}$$

$$D_f \cap D_g = \{n \mid f(n) = 0\} = \{\epsilon, 0\}$$

$$\frac{g}{f} = \left\{ \left(\epsilon, \frac{1}{\epsilon} \right), \left(0, 4 \right) \right\}$$

$$f(n) = \{(1, 2), (3, -1), (5, 2), (-1, 4)\}$$

$$g(n) = \{(-2, 0), (1, -1), (3, 2), (-1, 0)\}$$

انذ $f(2n) = \{(\frac{1}{2}, 2), (\frac{3}{2}, -1), (\frac{5}{2}, 2), (-\frac{1}{2}, 4)\}$

ب) $f(n^2) = \{(1, 2), (\sqrt{2}, -1), (2, 2), (-1, 4),$
 $(-\sqrt{2}, -1), (-2, 2)\}$

ج) $g(g(n) + 1) = \{(-2, \frac{10}{10}), (1, 9), (3, 9),$
 $(-1, 10)\}$

د) $\frac{f}{g} = \{(1, -2), (3, -1), \text{...}\}$

هـ) $f(n) = \{(2, 2), (0, 1), (4, 0), (-2, 0)\}$

$g(n) = \{(0, 1), (2, 0), (-2, 4), (-1, 2), (3, 2)\}$

$h = f + g$

$h = \{(2, 0), (2, 0), (-2, 4)\}$

د 1

$$f(n) = \frac{\sqrt{r^n - n}}{n - r}$$

$$g(n) = [n] + [-n]$$

$(\frac{f}{g+1}) | f(n)$ = $\{ (0, 0), (1, -\sqrt{r-1}), (r, 0) \}$

$$D_f \cap D_g = \{ n \mid f(n) \neq 0, g(n) = -1 \} = \{ 0, 1, r \}$$

$$D_f = [0, r] - \{ r \}$$

$$g(n) = R$$

$$n \geq 0$$

$$g(n) \neq -1$$

$$n \neq 0$$

$$\downarrow n \in \mathbb{Z}$$

$$\sqrt{r^n - n} \geq 0$$

$$\sqrt{r^n} \geq n$$

$$n \geq 0 \cup \downarrow$$

$$\forall n \geq r \rightarrow n^r - rn \leq 0$$

$$n(n-r) \leq 0$$

$$\frac{0}{r} \leq \frac{r}{r}$$

$$\textcircled{7}$$

$$f(n) = \sqrt{\omega n - n^r - 1}$$

$$g(n) = \{ (1, r), (r, r), (r, 1), (\omega, 1) \}$$

$$h(n) = \frac{f(n)}{g(n)}$$

$$\textcircled{1} D_f \cap D_g = \{ n \mid \omega n - n^r - 1 \geq 0, n^r - \omega n + 1 \leq 0 \} \rightarrow (n-1)(n-1) \leq 0 \quad \frac{1}{r} \leq \frac{r}{r}$$

$$D_f \cap D_g = \{1, 2, 4\}$$

$$h(n) = \left\{ (1, 0), \left(2, \frac{1}{2}\right), (4, 0) \right\}$$

v)

$$\text{انف } y = n^r - rn^r + rn + 1 - 1$$

$$y = (n-1)^r + 1$$

$$y-1 = (n-1)^r \rightarrow y \in \mathbb{R} \rightarrow R_f = \mathbb{R}$$

$$\sqrt[r]{y-1} = n-1$$

$$\text{ب) } y = \frac{1}{n^r - rn}$$

$$yn^r - rny = 1 \rightarrow yn^r - rny - 1 = 0$$

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$$\Delta \geq 0$$

$$2y^r + 2y \geq 0$$

$$y(y+1) \geq 0$$

$$\frac{-1}{2} \leq y \leq 0$$

$$R_f = (-\infty, -1] \cup [0, +\infty)$$

$$\text{(وژدو)} \quad y = \frac{1}{n^r - rn} \rightarrow n^r - rn = \frac{1}{y} \quad \text{بر دو طرف}$$

$$\frac{y+1}{y} \geq 0$$

$$\frac{-1}{2} \leq y \leq 0$$

$$n^r - rn + 1 = \frac{y+1}{y}$$

$$(n-1)^r = \frac{y+1}{y} \rightarrow n = \sqrt[r]{\frac{y+1}{y} + 1}$$

ا)

$$y = n^2 - 4n + 1$$

$$a > 0 \rightarrow \min | \frac{r}{4} \rightarrow R_f = [1, +\infty)$$

ب) $y = -n^2 + 4n + 7$

$$a < 0 \rightarrow \max | \frac{r}{1} \rightarrow R_f = (-\infty, 17]$$

ج) $y = \sqrt{n^2 - 2n - 3}$

$$n^2 - 2n - 3 \rightarrow a > 0 \rightarrow \min | \frac{r}{-1}$$

$$\downarrow [-1, +\infty) \rightarrow R_f = [0, +\infty)$$

بخط رادیکال

د)

د) $y = \sqrt{4n - n^2}$

$$4n - n^2 \rightarrow a < 0 \rightarrow \max | \frac{r}{1}$$

$$\downarrow [-\infty, 4] \rightarrow R_f = [0, 2]$$

بخط رادیکال

ه)

ه) $y = n^2 - 2n^2 + 2n + 1$

$$R_f = \mathbb{R}$$

و) $y = n^2 - 4n^2 + 4n + 7$

$$R_f = \mathbb{R}$$

$$2) y = \sqrt{n^2 - 4n^2 + 4n + 1}$$

$$\mathbb{R} \quad \downarrow \quad n^2 - 4n^2 + 4n + 1 \xrightarrow{2} \mathbb{R}$$

$$R_f = [0, +\infty)$$

بجای دیگر

$$3) y = (n^2 - 4n^2 + 4n + 1)^2$$

$$n^2 - 4n^2 + 4n + 1 \xrightarrow{2} \mathbb{R}$$

$$R_f = [0, +\infty)$$

بجای دیگر

1.1

$$الف) y = \frac{n+1}{n-2}$$

$$R_f = \mathbb{R} - \{2\}$$

$$ب) y = \frac{n+1}{n+1}$$

$$R_f = \mathbb{R} - \{2\}$$