

۱۹، ۲۵ آذین

الف) $y = x^2 - 4x + 10$

$x_{\min} = \frac{+4}{2} = 2 \quad y_{\min} = 4 - 16 + 10 = -2 \rightarrow R = [-2, +\infty)$

ب) $y = -x^2 + 4x + 3$

$x_{\max} = \frac{-4}{-2} = 2 \quad y_{\max} = -9 + 16 + 3 = 10 \rightarrow R = (-\infty, 10]$

ج) $y = \sqrt{x^2 - 4x}$

$x_{\min} = \frac{+4}{2} = 2 \quad y_{\min} = -4 \quad R = \sqrt{[-4, +\infty)} = [-2, +\infty)$

د) $y = \sqrt{4x - x^2}$

$x_{\max} = \frac{-4}{-2} = 2 \quad y_{\max} = 2 \quad R = \sqrt{[0, 4]} = [0, 2]$

الف) $y = x^3 - 6x^2 + 4x + 1$ در مجموع واصل $\rightarrow R_f = \mathbb{R}$

ب) $y = x^3 - 4x^2 + 3x + 2$ $\rightarrow R_f = \mathbb{R}$

ج) $y = \sqrt{x^3 - 4x^2 + 3x + 1}$ $\rightarrow R_f = \sqrt{\mathbb{R}} = [0, +\infty)$

د) $y = (x^3 - 4x^2 + 3x + 1)^2$ $\rightarrow R_f = [0, +\infty)$

الف) $y = \frac{x+1}{x-2}$ $\rightarrow R_f = \mathbb{R} - \{2\}$

ب) $y = \frac{x+1}{x+1}$ $\rightarrow R_f = \mathbb{R} - \{-1\}$

$$f(x) + g(x) = \frac{\sqrt{x+4}\sqrt{x-4}}{\sqrt{(\sqrt{x-4}+2)^2}} + \frac{\sqrt{x-4}\sqrt{x-4}}{\sqrt{(\sqrt{x-4}-2)^2}} \xrightarrow{x \geq 4} \frac{\sqrt{x-4} + \sqrt{x-4}}{\sqrt{x-4} + 2} + \frac{\sqrt{x-4} - \sqrt{x-4}}{\sqrt{x-4} - 2} = \frac{2\sqrt{x-4}}{\sqrt{x-4} + 2} - \frac{0}{\sqrt{x-4} - 2} = \frac{2\sqrt{x-4}}{\sqrt{x-4} + 2}$$

$$\rightarrow a=0 \quad b=2 \quad c=-4 \quad \frac{a+b}{c} = \frac{0+2}{-4} = -\frac{1}{2}$$

$$f(x) = \frac{x+2}{(x-1)(x-3)} \rightarrow D_f = \mathbb{R} - \{1, 3\}$$

$$\frac{g}{f} = \left\{ \left(\frac{1}{2}, \frac{1}{2} \right), \left(\frac{3}{2}, \frac{3}{2} \right) \right\}$$

$$f(2x) = \left\{ \left(\frac{1}{2}, 2 \right), \left(\frac{3}{2}, -1 \right), \left(\frac{5}{2}, 2 \right), \left(-\frac{1}{2}, 2 \right) \right\}$$

$$f(x^2) = \left\{ (1, 2), (\sqrt{3}, -1), (2, 2), (-1, 2), (-\sqrt{3}, -1), (-2, 2) \right\}$$

$$2g^2(x) + 1 = \left\{ (-2, 1), (1, 3), (2, 9), (-1, 1) \right\}$$

$$\frac{xf}{g} = \left\{ \left(1, \frac{4}{-1} \right), \left(2, -1 \right), \left(-\frac{1}{2}, \frac{2}{-1} \right) \right\}$$

من غن

$$h = 2f + g = \left\{ (2, 5), (-2, 5), (3, 5) \right\}$$

$$g(x) \xrightarrow{x \in \mathbb{Z}} 0$$

$$x \in \mathbb{Z} \rightarrow -1$$

$$\sqrt{x} - n \rightarrow \sqrt{x} \geq n \rightarrow x \geq n^2 \rightarrow x^2 - n^2$$

$$0 \leq n \leq \sqrt{x}, n \neq 2$$

$$f(x) = \frac{\sqrt{x^2 - x}}{x - 2} \xrightarrow{x \neq 2} \frac{0}{0} + \frac{1}{1} = 1$$

$$\rightarrow D\left(\frac{f}{g+1}\right) = \{x \mid x \in \mathbb{Z}, x \geq 4, x \neq 0\}$$

$$D\left(\frac{f}{g+1}\right) = \{0, 1, 3\}$$

$$\left(\frac{f}{g+1}\right)(x) \xrightarrow{x \neq -1} g \neq -1 \rightarrow x \notin \mathbb{Z}^+$$

$$f(x) = \sqrt{ax^2 - 2x - 4} = \sqrt{-(x-1)(x-4)} \rightarrow \frac{1}{-4+4} - \frac{4}{-4+4}$$

$$h(x) = \left\{ (1, 0), \left(\frac{\sqrt{5}}{2}, \frac{\sqrt{5}}{2} \right), (4, 0) \right\}$$

$$y = x^3 - 3x^2 + 3x + 1 - 1 = (x-1)^3 + 1 \rightarrow y-1 = (x-1)^3 \rightarrow x-1 = \sqrt[3]{y-1} \rightarrow x = \sqrt[3]{y-1} + 1$$

$$\rightarrow R_f = \mathbb{R}$$

$$y = \frac{1}{x^2 - 4x}$$

$$y(x^2 - 4x) = 1 \rightarrow yx^2 - 4yx - 1 = 0 \rightarrow x = \frac{4y \pm \sqrt{16y^2 + 4y}}{2y}$$

$$4y(y+1) \geq 0 \rightarrow \frac{-1}{-4} \leq y \leq \frac{1}{0} \rightarrow R = [-\frac{1}{4}, 0) \cup (0, +\infty)$$

$$f(n) = \frac{\sqrt{\sqrt{3n} - n}}{n-2} \rightarrow \begin{cases} n \neq 2 \\ n \geq 0 \end{cases} (\sqrt{3n}) \rightarrow Df = [0, 2] - \{2\} \quad (5)$$

$\sqrt{3n} - n \geq 0 \rightarrow 3n \geq n^2 \rightarrow 0 \leq n \leq 3$

$$g(n) + 1 \neq 0 \rightarrow g(n) \neq -1 \rightarrow [n] + [-n] \neq -1 \rightarrow n \in \mathbb{Z}$$

$$D \frac{f}{g+1} = \left\{ Df \cap Dg, g(n) \neq -1 \right\} = \left\{ 0, 1, 3 \right\}$$

$[0, 3] - \{2\} \cap \mathbb{Z}$

(الف) $y = n^3 - 3n^2 + 2n - 1 + 1 = y = (n-1)^3 + 1$ (6)

$$y-1 = (n-1)^3 \rightarrow \sqrt[3]{y-1} = n-1 \rightarrow n = \sqrt[3]{y-1} + 1$$

$$Rf = \mathbb{R}$$

(ب) $y = \frac{1}{n^2 - 2n} \rightarrow n^2 y - 2n y = 1 \rightarrow n^2 y - 2n y - 1 = 0$

$y \neq 0 \rightarrow n = \frac{2y \pm \sqrt{4y^2 + 4y}}{2y} \rightarrow 4y(y+1) \geq 0 \rightarrow \frac{-1}{+} \frac{0}{+}$

$y = 0 \rightarrow 0 = 1 \times$ صفتی وجود ندارد! $\rightarrow Rf = (-\infty, -1] \cup (0, +\infty)$