

۲. آزمون مدالی نوشت

$f(x) = \sqrt{x+4} + \sqrt{x-4} + 4 = \sqrt{(x-4)+8} = x-4+2 \xrightarrow{x \geq 4} \sqrt{x-4} + 2$ $g(x) = \sqrt{x-4} - \sqrt{x-4} + 4 = \sqrt{(x-4)-8} = \sqrt{x-4}-2 \xrightarrow{x \geq 4} g(x) = \sqrt{x-4} - 2$ $f(x) + g(x) = \sqrt{x-4} + 2 + \sqrt{x-4} - 2 = 2\sqrt{x-4}$ $f(x) + g(x) = a + b\sqrt{x+c}$ $\begin{cases} \rightarrow 2\sqrt{x-4} = a + b\sqrt{x+c} \rightarrow \begin{cases} a=0 \\ b=2 \\ c=-4 \end{cases} \\ \Rightarrow \frac{a+b}{c} = \frac{0+2}{-4} = -\frac{1}{2} \end{cases}$	۱
$f \neq 0 \rightarrow \begin{cases} x+2 \neq 0 \rightarrow x \neq -2 \\ x^2-4x+4 \neq 0 \rightarrow x \neq 1, 3 \end{cases} \Rightarrow D_f = \mathbb{R} - \{-2, 1, 3\}$ $D_g = \{2, 1, 3, 4\}$ $\Rightarrow \frac{g}{f} = \left\{ \left(2, \frac{1}{f}\right), \left(1, \frac{4}{f}\right) \right\} = \left\{ \left(2, -\frac{1}{4}\right), \left(0, 1\right) \right\}$	۲
۱) $\left\{ \left(2, \frac{1}{f}\right), \left(\frac{3}{f}, -1\right), \left(2, 2\right), \left(-\frac{1}{f}, 6\right) \right\}$ ۲) $\{ (1, 2), (-1, 2), (\sqrt{2}, -1), (-\sqrt{2}, -1), (2, 2), (-2, 2) \}$ ۳) $\{ (-2, 1), (1, 3), (2, 9), (-1, 1) \}$ ۴) $\{ (1, -4), (2, -1) \}$	۳
$D_h = \{2, -2, 3\}$ $h = \left\{ \left(2, \frac{1}{f}\right), \left(-2, \frac{1}{f}\right), \left(3, \frac{1}{f}\right) \right\} = \left\{ \left(2, 5\right), \left(-2, 6\right), \left(3, 4\right) \right\}$	۴
$f(x) = \frac{\sqrt{x}-x}{x-2} \rightarrow \begin{cases} \sqrt{x}-x \geq 0 \rightarrow x \leq \sqrt{x} \rightarrow x^2 \leq x \rightarrow x^2-x \leq 0 \rightarrow x(x-1) \leq 0 \rightarrow 0 \leq x \leq 1 \\ x-2 \neq 0 \rightarrow x \neq 2 \end{cases}$ $\Rightarrow D_{f(x)} = [0, 2) \cup (2, 1]$ $g(x) + 1 \neq 0 \rightarrow \begin{cases} \text{if } x=2 \rightarrow g(x)=0 \rightarrow g(x)+1=1 \\ \text{if } x \neq 2 \rightarrow g(x)=-1 \rightarrow g(x)+1=0 \end{cases} \Rightarrow D_g = \mathbb{Z}$	۵

$$\Rightarrow D\left(\frac{f}{g+1}\right)(x) = D_{f(x)} \cap D_{g(x)} = ([0, 2) \cup (2, 1]) \cap \mathbb{Z} \Rightarrow D\left(\frac{f}{g+1}\right)(x) = \{0, 1, 2\}$$

$f(x) = \sqrt{4x - x^2 - 5} \rightarrow -x^2 + 4x - 5 \geq 0 \rightarrow -(x-1)(x-5) \geq 0$ $\frac{1}{-x^2 + 4x - 5} \rightarrow D_f = [1, 5]$ $D_{g(x)} = \{1, 2, 5\}$ $\Rightarrow D_{h(x)} = D_{g(x)} \cap D_{f(x)} = \{1, 2, 5\}$ $h(x) = \{(1, 0), (2, \sqrt{3}), (5, 0)\}$	6
$\text{a)} y = x^2 - 2x^2 + 2x - 1 + 1 \rightarrow y = (x-1)^2 + 1 \rightarrow (x-1)^2 = y-1 \rightarrow x-1 = \sqrt{y-1}$ $\rightarrow x = \sqrt{y-1} + 1 \Rightarrow y \in \mathbb{R} \quad R_f = \mathbb{R}$ $\text{b)} y = \frac{1}{x^2 - 2x} \rightarrow yx^2 - 2yx = 1 \rightarrow yx^2 - 2y - 1 = 0$ $\rightarrow 4y^2 + 4y \geq 0 \rightarrow y^2 + y \geq 0 \rightarrow y(y+1) \geq 0$ $\frac{-1}{4} \leq y \leq 0 \rightarrow R_f = (-\infty, -1] \cup [0, +\infty)$	7
$\text{a)} y = x^2 - 4x + 1 \rightarrow \begin{cases} x_0 = -\frac{b}{2a} = \frac{4}{2} = 2 \\ y_0 = -4 \end{cases} \Rightarrow R = [4, +\infty)$ $\text{b)} y = -x^2 + 4x + 5 \rightarrow \begin{cases} x_0 = -\frac{b}{2a} = -\frac{4}{-2} = 2 \\ y_0 = -9 + 16 + 5 = 12 \end{cases} \Rightarrow R = (-\infty, 12]$ $\text{c)} } \begin{cases} x_0 = -\frac{b}{2a} = 2 \\ y_0 = -4 \end{cases} \rightarrow R_1 = \sqrt{[-4, +\infty)} = [0, +\infty)$ $\text{d)} } \begin{cases} x_0 = 2 \\ y_0 = -9 \end{cases} \rightarrow R_2 = \sqrt{[-9, 0]} = [0, 3]$	8
$\text{a)} y = (x^2 - 4x^2 + 2x + 1) \rightarrow R_f = \mathbb{R}$ $\text{b)} y = x^2 - 4x^2 + 2x + 1 \rightarrow R_f = \mathbb{R}$ $\text{c)} y = \sqrt{x^2 - 4x^2 + 2x + 1} \rightarrow R_f = \sqrt{1} = [0, +\infty)$ $\text{d)} y = (x^2 - 4x^2 + 2x + 1)^2 \rightarrow R_f = (\mathbb{R})^2 = [0, +\infty)$	9
$\text{a)} y = \frac{x+1}{x-2} \rightarrow R_f = \mathbb{R} - \left\{\frac{1}{2}\right\} = \mathbb{R} - \{2\}$ $\text{b)} y = \frac{x+1}{x+1} \rightarrow R_f = \mathbb{R} - \left\{\frac{1}{2}\right\} = \mathbb{R} - \{2\}$	10