

$$f(x) = \sqrt{x+4} + \sqrt{x-4} + 4 = \sqrt{(x-4)+8} = |x-4+2| \xrightarrow{x \geq 4} \sqrt{x-4} + 2$$

$$g(x) = \sqrt{x-4} - \sqrt{x-4} + 4 = \sqrt{(x-4)-8} = |\sqrt{x-4}-2| \xrightarrow{x \geq 4} g(x) = \sqrt{x-4} - 2$$

$$f(x) + g(x) = \sqrt{x-4} + 2 + \sqrt{x-4} - 2 = 2\sqrt{x-4}$$

$$f(x) + g(x) = a + b\sqrt{x+c}$$

$$\begin{cases} 2\sqrt{x-4} = a + b\sqrt{x+c} \rightarrow \begin{cases} a=0 \\ b=2 \\ c=-4 \end{cases} \\ \Rightarrow \frac{a+b}{c} = \frac{0+2}{-4} = -\frac{1}{2} \end{cases}$$

$$f \neq 0 \rightarrow \begin{cases} x+2 \neq 0 \rightarrow x \neq -2 \\ x^2-4x+4 \neq 0 \rightarrow x \neq 1, 3 \end{cases} \Rightarrow D_f = \mathbb{R} - \{-2, 1, 3\}$$

$$D_g = \{2, 1, 3, 4\}$$

$$\Rightarrow \frac{g}{f} = \left\{ \left(2, \frac{1}{2}\right), \left(1, \frac{1}{1}\right), \left(3, \frac{1}{3}\right) \right\} = \left\{ \left(2, -\frac{1}{2}\right), \left(0, 1\right) \right\}$$

$$1) \left\{ \left(2, \frac{1}{2}\right), \left(\frac{3}{2}, -1\right), \left(2, 2\right), \left(-\frac{1}{2}, 1\right) \right\}$$

$$2) \left\{ (1, 2), (-1, 2), (\sqrt{2}, -1), (-\sqrt{2}, -1), (2, 2), (-2, 2) \right\}$$

$$3) \left\{ (-2, 1), (1, 2), (2, 1), (-1, 1) \right\}$$

$$4) \left\{ (1, -4), (2, -1) \right\}$$

$$D_h = \{2, -2, 3\}$$

$$h = \left\{ \left(2, \frac{1}{2}\right), \left(-2, \frac{1}{2}\right), \left(3, \frac{1}{3}\right) \right\} = \left\{ (2, 2), (-2, 2), (3, 3) \right\}$$

$$f(x) = \frac{\sqrt{x}-x}{x-2} \rightarrow \begin{cases} \sqrt{x}-x \geq 0 \rightarrow x \leq \sqrt{x} \rightarrow x^2 \leq x \rightarrow x^2-x \leq 0 \rightarrow x(x-1) \leq 0 \rightarrow 0 \leq x \leq 1 \\ x-2 \neq 0 \rightarrow x \neq 2 \end{cases} \Rightarrow D_{f(x)} = [0, 2) \cup (2, 1]$$

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$$g(x) + 1 \neq 0 \rightarrow \begin{cases} \text{if } x = 2 \rightarrow g(x) = 0 \rightarrow g(x) + 1 = 1 \\ \text{if } x \neq 2 \rightarrow g(x) = -1 \rightarrow g(x) + 1 = 0 \end{cases} \Rightarrow D_g = \mathbb{Z}$$

$$\Rightarrow D_{\left(\frac{f}{g+1}\right)}(x) = D_{f(x)} \cap D_{g(x)} = ([0, 2) \cup (2, 1]) \cap \mathbb{Z} \Rightarrow D_{\left(\frac{f}{g+1}\right)}(x) = \{0, 1, 2\}$$

$f(x) = \sqrt{\Delta x - x^2 - \epsilon} \rightarrow -x^2 + \Delta x - \epsilon \geq 0 \rightarrow -(x-1)(x-\epsilon) \geq 0$ $-\frac{1}{x+1} \leq \frac{\epsilon}{x-1} \rightarrow D_f = [1, \epsilon]$ $D_{g_u} = \{1, 2, \epsilon, \Delta\}$ $\Rightarrow D_{h(x)} = D_{g(u)} \cap D_{f(u)} = \{1, 2, \epsilon\}$ $h(x) = \{(1, 0), (2, \frac{\sqrt{\Delta-1}}{2}), (\epsilon, 0)\}$	6
$\text{a)} y = x^2 - 2x^2 + 2x - 1 + 1 \rightarrow y = (x-1)^2 + 1 \rightarrow (x-1)^2 = y-1 \rightarrow x-1 = \sqrt{y-1}$ $\rightarrow x = \sqrt{y-1} + 1 \Rightarrow y \in \mathbb{R} \quad R_f = \mathbb{R}$ $\text{b)} y = \frac{1}{x^2 - 2x} \rightarrow yx^2 - 2yx = 1 \rightarrow yx^2 - 2y - 1 = 0 \rightarrow \Delta \geq 0, y \neq 0$ $\rightarrow 4y^2 + 4y \geq 0 \rightarrow y^2 + y \geq 0 \rightarrow y(y+1) \geq 0 \Rightarrow y \leq -1 \text{ or } y \geq 0$ $\Rightarrow R_f = (-\infty, -1] \cup [0, +\infty)$	7
$\text{a)} y = x^2 - 2x + 1 \rightarrow \begin{cases} x_0 = -\frac{b}{2a} = \frac{\epsilon}{2} = 1 \\ y_0 = 0 \end{cases} \Rightarrow R = [0, +\infty)$ $\text{b)} y = -x^2 + 2x + 2 \rightarrow \begin{cases} x_0 = -\frac{b}{2a} = -\frac{\epsilon}{2} = 1 \\ y_0 = 3 \end{cases} \Rightarrow R = (-\infty, 3]$ $\text{c)} } \begin{cases} x_0 = -\frac{b}{2a} = 1 \\ y_0 = -1 \end{cases} \rightarrow R_1 = \sqrt{[-1, +\infty)} = [0, +\infty)$ $\text{d)} } \begin{cases} x_0 = 1 \\ y_0 = -1 \end{cases} \rightarrow R_f = \sqrt{[-\infty, 1]} = [0, 1]$	8
$\text{a)} y = (x^2 - \Delta x^2 + 2x + 1) \rightarrow R_f = \mathbb{R}$ $\text{b)} y = x^2 - 2x^2 + 2x + 1 \rightarrow R_f = \mathbb{R}$ $\text{c)} y = \sqrt{x^2 - 2x^2 + 2x + 1} \rightarrow R_f = \sqrt{1} = [0, +\infty)$ $\text{d)} y = (x^2 - 2x^2 + 2x + 1)^2 \rightarrow R_f = (\mathbb{R})^2 = [0, +\infty)$	9
$\text{a)} y = \frac{x+1}{x-2} \rightarrow R_f = \mathbb{R} - \{\frac{1}{2}\} = \mathbb{R} - \{2\}$ $\text{b)} y = \frac{x+1}{x+1} \rightarrow R_f = \mathbb{R} - \{\frac{1}{2}\} = \mathbb{R} - \{2\}$	10