

نام و نام خانوادگی: محمد سید شریانی یا سخاوت شریانی کتبی شماره ۷ لاس الیسر A

$$f(x) = \sqrt{x+4}\sqrt{x-4} = \sqrt{x-4+4+4}\sqrt{x-4} = \sqrt{(\sqrt{x-4}+2)^2} = \sqrt{x-4}+2$$

$$g(x) = \sqrt{x-4}\sqrt{x-4} = \sqrt{x-4+4-4}\sqrt{x-4} = \sqrt{(\sqrt{x-4}-2)^2} = \sqrt{x-4}-2$$

$$f(x)+g(x) = \sqrt{x-4}+2+\sqrt{x-4}-2 = 2\sqrt{x-4} = a+b\sqrt{x}+c$$

$$a=0, b=2, c=-4 \Rightarrow \frac{a+b}{c} = \frac{0+2}{-4} = -\frac{1}{2} = \boxed{-\frac{1}{2}} \quad \text{د}$$

$$g(x) = \{(2,1), (0,0), (2,5), (0,4)\}$$

$$f(x) = \frac{x+5}{x^2-4x+5} \rightarrow x^2-4x+5 = (x-1)(x-5) = 0 \rightarrow \begin{matrix} x=1 \\ x=5 \end{matrix}$$

در تابع $f(x)$ در ازای $x=1$ و $x=5$ تعریف نشده است

$$x=5 \rightarrow \frac{5+5}{5-20+5} = \frac{10}{-10} = -1, g(5) = 1$$

$$x=0 \rightarrow \frac{0+5}{0-0+5} = 1, g(0) = 4 \rightarrow \frac{g}{f} = \{(2, \frac{1}{-1}), (0, \frac{4}{1})\}$$

$$\text{الف)} f(5x) = \{(\frac{1}{5}, 2), (\frac{5}{5}, -1), (2, 5), (-\frac{1}{5}, 6)\}$$

$$\text{ب)} f(x) = \{(\pm 1, 2), (\pm \sqrt{3}, -1), (\pm 2, 2)\}$$

$$\text{ج)} g(x) + 1 = \{(2, 19), (1, 5), (3, 9), (-1, 3)\}$$

$$\text{د)} \frac{f}{g} = \{(1, -4), (3, -1)\}$$

$$\begin{cases} x^2=1 \rightarrow x=\pm 1 \\ x^2=4 \rightarrow x=\pm 2 \\ x^2=3 \rightarrow x=\pm \sqrt{3} \end{cases}$$

$$g = \{(-2, 4), (2, 3), (3, 3)\}$$

$$f = \{(-2, 0), (2, 1), (3, 1)\}$$

$$h = 2f + g = \{(-2, 4), (2, 4), (3, 4)\}$$

$$f(x) = \frac{\sqrt{3x-x}}{x-2} \Rightarrow x \geq 0, x \neq 2, \sqrt{3x-x} \geq 0 \Rightarrow \sqrt{2x} \geq 0 \Rightarrow x \geq 0$$

$$\Rightarrow x \geq 0 \Rightarrow D_f = [0, 2] - \{2\} \quad g(x) \xrightarrow{2 \in \mathbb{Z}} [1] + [-1] = 0 \quad \checkmark$$

$$3 \notin \mathbb{Z} \rightarrow [1, 0] + [-1, 0] = -1 \quad \times$$

$$D_f = \{x | x \in \mathbb{Z}, 0 \leq x \leq 2, x \neq 2\} = \{0, 1, 2\}$$

$$f(x) = \sqrt{5x - x^2} - 4$$

$$f(0) = \sqrt{0 - 0} - 4 = -4$$

$$f(1) = \sqrt{5 - 1 - 4} = 0$$

$$f(2) = \sqrt{10 - 4 - 4} = \sqrt{2}$$

$$f(3) = \sqrt{15 - 9 - 4} = 0$$

$$f(4) = \sqrt{20 - 16 - 4} = -2$$

$$g(x) = \{(1, 5), (2, 4), (3, 3), (4, 2)\}$$

$$h(x) = \frac{f(x)}{g(x)} = \left\{ (1, 5), \left(2, \frac{\sqrt{2}}{4}\right), (4, 0) \right\}$$

$$y = x^2 - 3x^2 + 4x \quad (x-1)^2 = x^2 - 2x + 1 \quad y = (x-1)^2 + 1$$

$$\Rightarrow R_f = R$$

با فرض $y \neq 0$ عبارت را درجه ۲ فرض کنیم!

(ب) **حتمأً حاصل منفی است**

$$y = \frac{1}{x^2 - 2x} \Rightarrow yx^2 - 2xy = 1 \Rightarrow yx^2 - 2xy - 1 = 0$$

$$\Delta = b^2 - 4ac \Rightarrow (-2y)^2 - 4(-1)(y) = 4y^2 + 4y$$

$$\Rightarrow 4y^2 + 4y \geq 0 \Rightarrow 4y(y+1) \geq 0 \Rightarrow y(y+1) \geq 0 \Rightarrow y \in (-\infty, -1] \cup [0, +\infty)$$

$$y = -x^2 + 6x + 5 \Rightarrow y_5 = -\frac{\Delta}{4a}$$

$$36 - 4(-5) = 56 = \Delta \Rightarrow \frac{-6 \pm \sqrt{56}}{-2} = 15$$

$$R_f = (-\infty, 15]$$

$$y = x^2 - 4x + 1 \Rightarrow y_5 = -\frac{\Delta}{4a}$$

$$\Delta = b^2 - 4ac \Rightarrow 16 - 4 = 12 \Rightarrow y_5 = -\frac{12}{4} = -3$$

$$R_f = [6, +\infty)$$

$$y = \sqrt{4x - x^2} \quad \Delta = 16 - 4(-4) = 32$$

$$\frac{-2 \pm \sqrt{32}}{-2} = -4$$

$$R_f = [0, 4]$$

$$y = \sqrt{x^2 - 4x - 4} \quad \Delta = 16 - 4(-4) = 32$$

$$\frac{-2 \pm \sqrt{32}}{2} = 1$$

$$R_f = [1, +\infty)$$

$$y = x^2 - 4x^2 + 3x + 2$$

$$R_f = R$$

$$y = x^2 - 5x^2 + 2x + 1$$

$$R_f = R$$

$$y = (x^2 - 4x^2 + 3x + 1)^2$$

$$R = R$$

$$(R)^c = [0, +\infty) \Rightarrow R_f = [0, +\infty)$$

$$y = \sqrt{x^2 - 4x^2 + 3x + 1}$$

$$R = R$$

$$\sqrt{R} = [0, +\infty) \Rightarrow R_f = [0, +\infty)$$

(الف)

$$y = \frac{2x+1}{x-2} \Rightarrow xy - 2y = 2x+1 \Rightarrow 2x+1 - xy + 2y = 0$$

$$\Rightarrow x(2-y) + 1 + 2y \Rightarrow x(2-y) = -2y-1 \Rightarrow x = \frac{-2y-1}{2-y} \Rightarrow 2-y \neq 0$$

$$\Rightarrow y \neq 2 \Rightarrow R_f = R \setminus \{2\}$$

(ب)

$$y = \frac{2x+1}{x+1} \Rightarrow xy + y = 2x+1 \Rightarrow 2x+1 - xy + y = 0$$

$$\Rightarrow x(2-y) + 1 + y \Rightarrow x(2-y) = -y-1 \Rightarrow x = \frac{-y-1}{2-y} \Rightarrow 2-y \neq 0$$

$$\Rightarrow y \neq 2 \Rightarrow R_f = R \setminus \{2\}$$

(۳) x های f باید حتمی برابر شوند! $f(2n) = \left\{ \left(\frac{1}{2}, 2\right) \left(\frac{3}{2}, -1\right) (2, 2) \left(-\frac{1}{2}, 4\right) \right\}$

هر کدام از x های f را برابر n^2 قرار دهیم! $f(2) = \left\{ (1, 2) (-1, 2) (\sqrt{3}, -1) (-\sqrt{3}, -1) (2, 2) (-2, 2) \right\}$

این عمل روی y های و اعمال می شود: $f_2(n) + 1 = \left\{ (-2, 19) (1, 3) (3, 9) (-1, 1) \right\}$

$$\frac{f_2}{g} = \left\{ \left(1, \frac{2 \times 2}{-1}\right) \left(3, \frac{2 \times -1}{2}\right) \text{ و } \left(-1, \frac{2 \times 4}{-}\right) \right\} = \left\{ (1, -2) (3, -1) \right\}$$

(الف) $y = n^3 - 2n^2 + 2n - 1 + 1 = y = (n-1)^3 + 1$ (۷)

$$y - 1 = (n-1)^3 \rightarrow \sqrt[3]{y-1} = n-1 \rightarrow n = \sqrt[3]{y-1} + 1$$

$R_f = \mathbb{R}$

(ب) $y = \frac{1}{n^2 - 2n} \rightarrow n^2 y - 2n y = 1 \rightarrow n^2 y - 2n y - 1 = 0$

$y \neq 0 \rightarrow n = \frac{2y \pm \sqrt{4y^2 + 4y}}{2y} \rightarrow 4y(y+1) \geq 0 \rightarrow \frac{-1}{+} \frac{0}{+}$

$y = 0 \rightarrow 0 = 1 \times$ صحتی وجود ندارد! $\rightarrow R_f = (-\infty, -1] \cup (0, +\infty)$

$$\text{الف - ١)} \quad y = x^2 - 2x + 1 \rightarrow \min \left| \begin{array}{l} x = \frac{2}{2} = 1 \\ y = 1 - 2 + 1 = 0 \end{array} \right. \rightarrow \text{Rf} = [0, +\infty)$$

$$\text{ب - ١)} \quad y = -x^2 + 4x + 3 \rightarrow \max \left| \begin{array}{l} x = \frac{-4}{-2} = 2 \\ y = -9 + 18 + 3 = 12 \end{array} \right. \rightarrow \text{Rf} = (-\infty, 12]$$

$$\text{ج - ١)} \quad y = x^2 - 2x - 3 \rightarrow \min \left| \begin{array}{l} x = \frac{2}{2} = 1 \\ y = 1 - 2 - 3 = -4 \end{array} \right. \rightarrow \text{Rf} = \sqrt{[-4, +\infty)} = [-4, +\infty)$$

$$\text{د - ١)} \quad y = 4x - x^2 \rightarrow \max \left| \begin{array}{l} x = \frac{-4}{-2} = 2 \\ y = 18 - 4 = 14 \end{array} \right. \rightarrow \text{Rf} = \sqrt{(-\infty, 14]} = (-\infty, 14]$$