

۲. آزمون ادبی و کامل منصفیت

$$\begin{aligned} \varphi(x) &= \sqrt{x+4\sqrt{x-4}} & \varphi(x) &= \sqrt{(\sqrt{x-4}+2)^2} = |\sqrt{x-4}+2| = \sqrt{x-4}+2 \\ g(x) &= \sqrt{x-4\sqrt{x-4}} & g(x) &= \sqrt{(\sqrt{x-4}-2)^2} = |\sqrt{x-4}-2| \rightarrow \sqrt{x-4}-2 \\ \varphi(x)+g(x) &= a+b\sqrt{x+c} \end{aligned}$$

چون $x \gg 8$ ، $\sqrt{x-4} \approx \sqrt{x}$ ، $\sqrt{x-4}-2 \approx \sqrt{x}-2$ ، $\sqrt{x-4}+2 \approx \sqrt{x}+2$

$$\varphi(x)+g(x) = (\sqrt{x-4}+2) + (\sqrt{x-4}-2) = 2\sqrt{x-4} \Rightarrow \begin{cases} a=0 \\ b=2 \\ c=-4 \end{cases}$$

$\frac{a+b}{c} = \frac{0+2}{-4} = -\frac{1}{2}$

$$\begin{aligned} \varphi(x) &= \frac{x+2}{x^2-4x+3} \rightarrow \varphi(x) = \frac{x+2}{(x-1)(x-3)} \Rightarrow D_\varphi = \mathbb{R} - \{1, 3\} & D_{\frac{g}{\varphi}} &= \{0, 2\} \\ g(x) &= \{ (2, 1), (1, 1), (0, 1), (3, 0), (1, 0), (2, 1) \} \Rightarrow D_g = \{0, 1, 2, 3\} & D_{\frac{g}{\varphi}} &= D_g \cap D_\varphi = \{x \mid \varphi(x) \neq 0\} \\ & \text{مونده های اول} & & \\ g(0) &= 1 & \varphi(0) &= \frac{2}{3} \Rightarrow \frac{g(0)}{\varphi(0)} = \frac{3}{2} \\ g(2) &= 1 & \varphi(2) &= -1 \Rightarrow \frac{g(2)}{\varphi(2)} = -1 \end{aligned}$$

$$\frac{g}{\varphi} = \{ (0, \frac{3}{2}), (2, -1) \}$$

$$\begin{aligned} \varphi(x) &= \{ (1, 2), (3, -1), (4, 2), (-1, 2) \} & g(x) &= \{ (-2, 3), (1, -1), (3, 2), (1, 1) \} & D_\varphi &= \{-1, 2, 3, 4\} \\ & & & & D_g &= \{-2, 1, 3, 4\} \\ \text{الف) } \varphi(2x) &= \{ (-\frac{1}{2}, 2), (2, 2), (\frac{3}{2}, -1), (\frac{1}{2}, 2) \} \leftarrow 2x \in D_\varphi \\ \text{ب) } \varphi(x^2) &= \{ (2, 2), (-2, 2), (\sqrt{3}, -1), (-\sqrt{3}, -1), (1, 2), (-1, 2) \} \leftarrow x^2 \in D_\varphi \\ \text{ج) } 2g^2(x)+1 &= \{ (-1, 1), (3, 9), (1, 3), (-2, 19) \} \leftarrow \text{هر وجهی را برای هر عضوهای } D \text{ در اینجا دامنه های } D \text{ هست فقط باید صحت } \\ \text{د) } \frac{2\varphi}{g} &= \{ (1, -4), (3, -1) \} & D_\varphi \cap D_g &= \{-1, 2, 3\} & \text{چون } g(-1) &= 0 \text{ است پس } x = -1 \text{ حذف می شود} \end{aligned}$$

$$\begin{aligned} \varphi(x) &= \{ (2, 1), (3, 1), (4, 9), (-2, 0) \} & g &= \{ (5, 1), (2, 3), (-2, 7), (-1, 2), (3, 3) \} \\ h &= 2\varphi + g & D_\varphi &= \{2, 3, 4, -2\} & D_g &= \{5, 2, -2, -1, 3\} & D_h &= D_\varphi \cap D_g = \{2, 3, -2\} \\ h(2) &= 2\varphi(2) + g(2) = 5 & & & & & \\ h(3) &= 2\varphi(3) + g(3) = 5 & \Rightarrow h &= \{ (2, 5), (3, 5), (-2, 7) \} & & & \\ h(-2) &= 2\varphi(-2) + g(-2) = 7 & & & & & \end{aligned}$$

$$\begin{aligned} \varphi(x) &= \frac{\sqrt{3x-x}}{x-2} & g(x) &= [x] + [-x] = \begin{cases} 0 & x \in \mathbb{Z} \\ -1 & x \notin \mathbb{Z} \end{cases} & g(x) \neq -1 &\Rightarrow x \in \mathbb{Z} \\ 3x > 0 &\Rightarrow x > 0 \\ \sqrt{3x-x} > 0 &\Rightarrow \sqrt{3x} > x \\ \rightarrow x^2 - 3x < 0 &\Rightarrow 0 < x < 3 \\ x &\neq 2 \end{aligned}$$

$$\Rightarrow D_\varphi = [0, 3] - \{2\}$$

$$x \in ([0, 3] - \{2\}) \cap \mathbb{Z} \Rightarrow x \in \{0, 1, 3\}$$

$$D = \{0, 1, 3\}$$

$\varphi(x) = \sqrt{ax - x^2 - f}$ $g(x) = \{(1, 1), (2, 2), (4, 7), (5, 1)\}$ $h(x) = \frac{\varphi(x)}{g(x)}$ $D_g = \{1, 2, 4, 5\}$ $D_h = D_\varphi \cap D_g = \{1, 2, 4\}$ $ax - x^2 - f \geq 0 \rightarrow x^2 - ax + f \leq 0$ $\Rightarrow 1 \leq x \leq 4 \Rightarrow D_\varphi = [1, 4]$ $\varphi(1) = 0 \Rightarrow h(1) = 0$ $g(1) = 1$ $\varphi(2) = \sqrt{2} \Rightarrow h(2) = \frac{\sqrt{2}}{2}$ $g(2) = 2$ $\varphi(4) = 0 \Rightarrow h(4) = 0$ $g(4) = 7$ $h = \{(1, 0), (2, \frac{\sqrt{2}}{2}), (4, 0)\}$	
الف) $y = x^3 - 3x^2 + 3x$ $x - 1 = \sqrt[3]{y - 1} \Rightarrow x = \sqrt[3]{y - 1} + 1$ ب) $y = \frac{1}{x^2 - 2x}$ $yx^2 - 2yx - 1 = 0 \Delta \geq 0 \Rightarrow 4y^2 + 4y \geq 0 \Rightarrow y(y + 1) \geq 0$ $\Rightarrow y \leq -1 \text{ or } y \geq 0$ $R_\varphi = (-\infty, -1] \cup [0, +\infty)$	
الف) $y = x^2 - 4x + 1$ $x_s = -\frac{b}{2a} = -\frac{-4}{2} = 2$ $y_s = 2^2 - 4 \cdot 2 + 1 = -7$ $R_\varphi = [-7, +\infty)$ ب) $y = -x^2 + 7x + 3$ $x_s = -\frac{b}{2a} = -\frac{7}{-2} = \frac{7}{2}$ $y_s = -(\frac{7}{2})^2 + 7 \cdot \frac{7}{2} + 3 = 12$ $R_\varphi = (-\infty, 12]$ ج) $y = \sqrt{x^2 - 4x - 3}$ $x_s = -\frac{b}{2a} = -\frac{-4}{2} = 2$ $y_s = \sqrt{2^2 - 4 \cdot 2 - 3} = \sqrt{-5}$ $R_\varphi = [0, +\infty)$ د) $y = \sqrt{7x - x^2}$ $x_s = -\frac{b}{2a} = -\frac{-7}{-2} = \frac{7}{2}$ $y_s = \sqrt{7 \cdot \frac{7}{2} - (\frac{7}{2})^2} = \sqrt{\frac{49}{4} - \frac{49}{4}} = 0$ $R_\varphi = [0, 9]$	
الف) $y = x^3 - 5x^2 + 4x + 1$ $R_\varphi = \mathbb{R}$ ب) $y = x^3 - 7x^2 + 3x + 2$ $R_\varphi = \mathbb{R}$ ج) $y = \sqrt{x^3 - 7x^2 + 5x + 1}$ $R_\varphi = [0, +\infty)$ د) $y = (x^3 - 4x^2 + 3x + 1)^2$ $R_\varphi = [0, +\infty)$	
الف) $y = \frac{3x+1}{x-2}$ $R_y = \mathbb{R} - \{\frac{a}{c}\} \Rightarrow R_y = \mathbb{R} - \{\frac{1}{-2}\}$ ب) $x = -\frac{-2y-1}{y-3} \Rightarrow x = \frac{2y+1}{y-3} \Rightarrow R_y = \mathbb{R} - \{3\}$ ج) $y = \frac{3x+1}{x+1} \Rightarrow R_y = \mathbb{R} - \{\frac{a}{c}\} \Rightarrow R_y = \mathbb{R} - \{\frac{1}{1}\}$ د) $x = -\frac{y-1}{y-2} \Rightarrow R_y = \mathbb{R} - \{2\}$	1.