

$$\begin{aligned} \varphi(x) &= \sqrt{x-4}\sqrt{x-4} & \varphi(x) &= \sqrt{(\sqrt{x-4}+2)^2} = |\sqrt{x-4}+2| = \sqrt{x-4}+2 \\ g(x) &= \sqrt{x-4}\sqrt{x-4} & g(x) &= \sqrt{(\sqrt{x-4}-2)^2} = |\sqrt{x-4}-2| \rightarrow \sqrt{x-4}-2 \\ \varphi(x)+g(x) &= a+b\sqrt{x+c} \\ \varphi(x)+g(x) &= (\sqrt{x-4}+2) + (\sqrt{x-4}-2) = 2\sqrt{x-4} \Rightarrow \begin{cases} a=0 \\ b=2 \\ c=-4 \end{cases} \end{aligned}$$

$\frac{a+b}{c} = \frac{0+2}{-4} = -\frac{1}{2}$

$$\begin{aligned} \varphi(x) &= \frac{x+2}{x^2-4x+3} \rightarrow \varphi(x) = \frac{x+2}{(x-1)(x-3)} \Rightarrow D_\varphi = \mathbb{R} - \{1, 3\} & D_{\frac{g}{\varphi}} &= \{0, 2\} \\ g(x) &= \{(1, 1), (1, 0), (3, 0), (0, 2)\} \Rightarrow D_g = \{0, 1, 2, 3\} & D_{\frac{g}{\varphi}} &= D_g \cap D_\varphi = \{x \mid \varphi(x) \neq 0\} \\ \frac{g}{\varphi} &= \{(0, 2), (2, -\frac{1}{2})\} \end{aligned}$$

$\frac{g(0)}{\varphi(0)} = \frac{2}{1} = 2$ $\frac{g(2)}{\varphi(2)} = \frac{-1}{2} = -\frac{1}{2}$

$$\begin{aligned} \varphi(x) &= \{(1, 2), (3, -1), (4, 2), (-1, 2)\} & g(x) &= \{(1, 2), (1, -1), (3, 2), (-1, 1)\} & D_\varphi &= \{-1, 2, 3, 4\} \\ \text{الف) } \varphi(2x) &= \{(-\frac{1}{2}, 2), (2, 2), (\frac{3}{2}, -1), (\frac{1}{2}, 2)\} \leftarrow 2x \in D_\varphi & D_g &= \{-1, 2, 1, -1\} \\ \text{ب) } \varphi(x^2) &= \{(2, 2), (-2, 2), (\sqrt{3}, -1), (-\sqrt{3}, -1), (1, 2), (-1, 2)\} \leftarrow x^2 \in D_\varphi \\ \text{ج) } 2g^2(x)+1 &= \{(-1, 1), (3, 9), (1, 3), (-2, 19)\} \leftarrow \text{در اینجا دامنه همان } D_g \text{ است فقط باید متوجه شویم} \\ \text{د) } \frac{2\varphi}{g} &= \{(1, -4), (3, -1)\} & D_\varphi \cap D_g &= \{-1, 2, 1\} \end{aligned}$$

چون $g(-1)=0$ و $x=-1$ حذف می شود

$$\begin{aligned} \varphi(x) &= \{(2, 1), (3, 1), (4, 9), (-2, 0)\} & g &= \{(5, 1), (2, 3), (-2, 7), (-1, 2), (3, 3)\} \\ h &= 2\varphi + g & D_\varphi &= \{2, 3, 4, -2\} & D_g &= \{5, 2, -2, -1, 3\} & D_h &= D_\varphi \cap D_g = \{2, 3, -2\} \\ h(2) &= 2\varphi(2) + g(2) = 5 \\ h(3) &= 2\varphi(3) + g(3) = 5 \\ h(-2) &= 2\varphi(-2) + g(-2) = 7 \end{aligned}$$

$\Rightarrow h = \{(2, 5), (3, 5), (-2, 7)\}$

$$\begin{aligned} \varphi(x) &= \frac{\sqrt{3x-x}}{x-2} & g(x) &= [x] + [-x] = \begin{cases} 0 & x \in \mathbb{Z} \\ -1 & x \notin \mathbb{Z} \end{cases} & g(x) \neq -1 &\Rightarrow x \in \mathbb{Z} \\ 3x > 0 &\Rightarrow x > 0 \\ \sqrt{3x-x} > 0 &\Rightarrow \sqrt{3x} > x \\ \rightarrow x^2 - 3x < 0 &\Rightarrow 0 < x < 3 \\ x &\neq 2 \end{aligned}$$

$\Rightarrow D_\varphi = [0, 3] - \{2\}$ $x \in ([0, 3] - \{2\}) \cap \mathbb{Z} \Rightarrow x \in \{0, 1, 3\}$ $D = \{0, 1, 3\}$

$\varphi(x) = \sqrt{ax - x^2 - f}$ $D_g = \{1, 2, 4, \omega\}$ $\omega x - x^2 - f \geq 0 \rightarrow x^2 - \omega x + f \leq 0$ $\Rightarrow 1 \leq x \leq 4 \Rightarrow D_\varphi = [1, 4]$	$g(x) = \{(1, 2), (2, 4), (4, 7), (\omega, 1)\}$ $D_h = D_\varphi \cap D_g = \{1, 2, 4\}$ $h(x) = \frac{\varphi(x)}{g(x)}$ $\varphi(1) = 0, g(1) = 2 \Rightarrow h(1) = 0$ $\varphi(2) = \sqrt{2}, g(2) = 4 \Rightarrow h(2) = \frac{\sqrt{2}}{4}$ $\varphi(4) = 0, g(4) = 7 \Rightarrow h(4) = 0$ $h = \{(1, 0), (2, \frac{\sqrt{2}}{4}), (4, 0)\}$
$\text{الف) } y = x^3 - 3x^2 + 3x$ $x - 1 = \sqrt[3]{y - 1} \Rightarrow x = \sqrt[3]{y - 1} + 1$ $\text{ب) } y = \frac{1}{x^2 - 2x}$ $yx^2 - 2yx - 1 = 0 \Delta \geq 0 \Rightarrow 4y^2 + 4y \geq 0 \Rightarrow y(y + 1) \geq 0$ $\Rightarrow y \leq -1 \text{ or } y \geq 0$ $R_\varphi = (-\infty, -1] \cup (0, +\infty)$	$y - 1 = x^3 - 3x^2 + 3x - 1 \Rightarrow y - 1 = (x - 1)^3 \Rightarrow y \in \mathbb{R}, R_\varphi = \mathbb{R}$
$\text{الف) } y = x^2 - 4x + 1$ $x_s = -\frac{b}{2a} = -\frac{-4}{2} = 2, y_s = 4 - 8 + 1 = -3 \Rightarrow R_\varphi = [-3, +\infty)$ $\text{ب) } y = -x^2 + 7x + 3$ $x_s = -\frac{b}{2a} = -\frac{7}{-2} = \frac{7}{2}, y_s = -\frac{49}{4} + \frac{49}{2} + 3 = \frac{17}{4} \Rightarrow R_\varphi = (-\infty, \frac{17}{4}]$ $\text{ج) } y = \sqrt{x^2 - 4x - 3}$ $x_s = -\frac{b}{2a} = -\frac{-4}{2} = 2, y_s = 4 - 8 - 3 = -7 \Rightarrow y = \sqrt{(-\infty, -7]}$ $\Rightarrow x \leq 3, R_\varphi = [-7, +\infty)$	$\Rightarrow R_\varphi = [0, +\infty)$
$\text{الف) } y = x^3 - 5x^2 + 4x + 1$ $\text{ب) } y = x^3 - 7x^2 + 3x + 2$ $\text{ج) } y = \sqrt{x^3 - 7x^2 + 5x + 1}$ $\text{د) } y = (x^3 - 4x^2 + 3x + 1)^2$	$\rightarrow R_\varphi = \mathbb{R}$ $\rightarrow R_\varphi = \mathbb{R}$ $\rightarrow R_\varphi = \sqrt{\mathbb{R}} \Rightarrow R_\varphi = [0, +\infty)$ $\rightarrow R_\varphi = (\mathbb{R})^2 \Rightarrow R_\varphi = [0, +\infty)$
$\text{الف) } y = \frac{3x+1}{x-2}$ $x = -\frac{-2y-1}{y-3} \Rightarrow x = \frac{2y+1}{y-3} \Rightarrow R_y = \mathbb{R} - \{3\}$ $\text{ب) } y = \frac{2x+1}{x+1}$ $x = -\frac{y-1}{y-2} \Rightarrow x = \frac{1-y}{y-2} \Rightarrow R_y = \mathbb{R} - \{2\}$	$\Rightarrow R_y = \mathbb{R} - \{2\}$