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 $f(x) = \sqrt{x+1} \sqrt{x-1}$, $g(x) = \sqrt{x-1} \sqrt{x+1}$ $\rightarrow \sqrt{(x-1)} = a$
 $\rightarrow (a-1)^2 = (1-1)^2 \rightarrow \sqrt{x-1} = \sqrt{(a^2+1)-1} = \sqrt{a^2} = a$
 ۱) $(a^2+1)+1 = \sqrt{a^2+1} + \sqrt{a^2+1} = 2\sqrt{a^2+1} = 2(a+1)$
 $f(x) = a+1$
 $f(x)+g(x) = 2a = 2\sqrt{x-1}$
 $a+b\sqrt{x+c} = 2\sqrt{x-1} \rightarrow \frac{a+b}{c} = \frac{2}{-1} = -2$

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 $f(x) = \frac{x+1}{x^2-x+1} \rightarrow f(1) = -1$, $g(1) = 1 \rightarrow (g/f)(1) = -1$
 $(x-1)(x+1) f(x) = 1 \rightarrow g(x) = 1 \rightarrow (g/f)(x) = 1$
 $\frac{g}{f} = \{1, -1, 0, 1\}$

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 الف) $f(x) = \{(1, 1), (1, -1), (1, 1), (-1, 1)\}$
 ب) $f(x) = \{(1, 1), (-1, 1), (\sqrt{2}, -1), (-\sqrt{2}, -1), (1, 1), (-1, 1)\}$
 ج) $2g(x)+1 \rightarrow \{(-1, 1), (1, 1), (1, 1), (-1, 1)\}$
 د) $2f/g \rightarrow \{(1, -1), (1, -1)\}$

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 $h = 2fg \rightarrow h(x) = 2(1)+1 = 3$
 $h(x) = 2(1)+1 = 3 \rightarrow h = \{(1, 3), (1, 3), (-1, 3)\}$
 $f \cap g = \{(1, 1), (-1, 1)\}$, $h(x) = 2(0)+1 = 1$

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 $\frac{f}{g} + 1 \rightarrow g+1 \neq 0 \rightarrow g+1=1$
 $g \geq 0$, $x-x^2 \geq 0 \rightarrow x \in [0, 1]$
 $x-1 \neq 0 \rightarrow x \neq 1$
 $f \cap g \rightarrow \{0, 1\} \rightarrow \{0, 1\}$
 $\rightarrow D \{0, 1\}$

$$f(x) = \sqrt{x-x^2-\epsilon^2} \rightarrow (x-1)(x-\epsilon) \leq 0 \rightarrow x \in [1, \epsilon]$$

$$h(x) = \frac{f(x)}{g(x)}$$

$$x=1 \rightarrow f(1)=0 \quad g(1)=1 \rightarrow h=0$$

$$x=\epsilon \rightarrow f(\epsilon)=\sqrt{\epsilon} \quad g(\epsilon)=\epsilon \rightarrow h=\frac{\sqrt{\epsilon}}{\epsilon}$$

$$x=\epsilon \rightarrow f(\epsilon)=0 \quad g(\epsilon)=\epsilon \rightarrow h=0$$

$$h = \{(1,0), (\epsilon, \frac{\sqrt{\epsilon}}{\epsilon}), (\epsilon, 0)\}$$

$$y = x^r - r x^{r-1} = (x-1)^r \rightarrow \mathbb{R}$$

$$\therefore y = \frac{1}{x^{r-1}} \rightarrow x^{r-1} = (x-1)^{r-1} \rightarrow t = (x-1)^{r-1} \rightarrow [-1, 0) \cup (0, +\infty)$$

$$\searrow y = \frac{1}{t} \rightarrow (-\infty, -1] \cup (0, +\infty)$$

$$y = x^{r-\epsilon} + 10 = (x-r)^r + 1 \rightarrow \mathbb{R} : [1, +\infty)$$

$$\therefore y = -x^r + r x^{r-1} = -(x-r)^r + 1 \rightarrow \mathbb{R} : (-\infty, 1]$$

$$c) y = \sqrt{x^r - \epsilon x^{r-1}} = \sqrt{(x-r)^r - 1} \rightarrow \mathbb{R} : [0, \infty)$$

$$d) y = \sqrt{x^r - x^r} = \sqrt{1 - (x-r)^r} \rightarrow [0, r]$$

~~more examples~~

$$y = x^r - 2x^r + r x^{r+1} \rightarrow \mathbb{R} : \mathbb{R}$$

$$y = x^r - r x^{r-1} + r x^{r-1} \rightarrow \mathbb{R} : \mathbb{R}$$

$$y = \sqrt{(x^r - r x^{r-1} + r x^{r-1})} \rightarrow \mathbb{R} : [0, +\infty)$$

$$y = (x^r - \epsilon x^{r-1} + r x^{r-1})^r \rightarrow \mathbb{R} : [0, +\infty)$$

$$y = \frac{(r x^{r+1})}{x-r} \rightarrow a=r, c=1 \rightarrow \mathbb{R} : \mathbb{R} - \{r\}$$

$$y = \frac{(r x^{r+1})}{x+1} \rightarrow a=r, c=1 \rightarrow \mathbb{R} : \mathbb{R} - \{-1\}$$