

$$A = \{1, 2, 3\} \quad B = \{2, 3\}$$

$$A \times B - B' = \{(1, 2), (1, 3), (2, 2), (2, 3), (3, 2), (3, 3)\} - \{(2, 2), (2, 3), (3, 2), (3, 3)\}$$

$$= \{(1, 2), (1, 3)\}$$

الف) $f(x) = \{(3, m^2), (2, 1), (-3, m), (-2, m), (3, m+1), (m, 4)\}$

$$\Rightarrow (3, m^2) \text{ و } (3, m+1) \Rightarrow m^2 = m+1 \rightarrow m^2 - m - 1 = 0 \quad (m-2)(m+1) = 0$$

غیر ممکن $m=2$
غیر ممکن $m=-1$

ب) $g(x) = \{(3, m^2), (2, 1), (m, 4), (3, m+1), (-2, m), (-1, 3)\}$

$$\Rightarrow (3, m^2) \text{ و } (3, m+1) \rightarrow m^2 = m+1 \rightarrow m^2 - m - 1 = 0 \rightarrow (m-2)(m+1) = 0$$

غیر ممکن $m=2$
غیر ممکن $m=-1$

$$f(x) = \begin{cases} x^2 - 1 & ; x \leq 1 \\ ax + b & ; 1 < x \leq 2 \\ x^3 & ; x > 2 \end{cases}$$

(I) $x^2 - 1 = ax + b \xrightarrow{x=1} a + b = 0 \quad \boxed{a = -b}$

(II) $x^3 = ax + b \xrightarrow{x=2} 8 = 2a + b \quad \begin{cases} 2a + b = 8 \\ a = -b \end{cases} \Rightarrow -2b + b = 8 \Rightarrow -b = 8 \Rightarrow b = -8 \Rightarrow a = 8$

$$\Rightarrow a \times b = 8 \times -8 = -64$$

$$f(x) = \begin{cases} x^2 - 3 & ; x \geq k \\ 4 - 3x & ; x \leq k \end{cases}$$

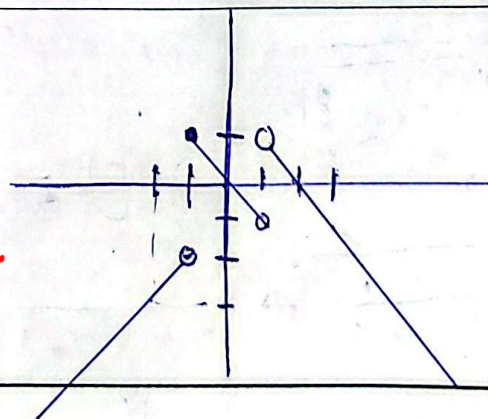
$f(k) = f(k) \xrightarrow{x=k} k^2 - 3 = 4 - 3k \quad \Delta k = 7$

$$\Rightarrow k = \frac{7}{2} = 1, 4$$

$$f(x) = \begin{cases} -x + 1 & ; x > 1 \\ -x & ; -1 \leq x < 1 \\ x - 1 & ; x < -1 \end{cases}$$

$$D_f = (-\infty, +1) \cup (1, +\infty)$$

$$R_f = (-\infty, 1]$$



الف) $y^3 + y^2 + 1 = 0 \rightarrow y_1^3 = -y_1^2 - 1$ $y_1^3 = y_2^3 \Rightarrow y_1 = y_2$ تابع است ✓

ب) $x^2 + y^2 - 2x - 12y + 10 = 0 \rightarrow x^2 - 2x + 1 + y^2 - 12y + 1 = (x-1)^2 + (y-6)^2 = 0$
 $\Rightarrow \begin{cases} x=1 \\ y=6 \end{cases}$ فقط در این نقطه متعلق می باشد ✓

الف) $x \sin y = y \sin x \xrightarrow{x=0} y = \mathbb{R} \Rightarrow$ اگر x صفر باشد y هر عددی می تواند باشد پس تابع نیست ✓

ب) $\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = 2 \Rightarrow \begin{cases} x=1 \\ y=1 \end{cases}$ فقط در این نقطه متعلق می باشد
 به ازای تمام $x=y$ ها تابع می باشد ✓

الف) $y = \sqrt{\frac{x-1}{x-3}} + \sqrt{\frac{2-x}{x}}$ $\frac{x-1}{x-3} \geq 0 \rightarrow \frac{1}{1+\phi-\phi} + \frac{2-x}{x} \geq \frac{0}{1-\phi+\phi}$
 $\Rightarrow D_f = (0, 1]$ ✓

ب) $\sqrt{x} + \sqrt{y-1} = \sqrt{3} \rightarrow (\sqrt{y-1})^2 = (\sqrt{3}-\sqrt{x})^2 \rightarrow y \geq 1 \rightarrow \sqrt{3}-\sqrt{x} \geq 0 \rightarrow x \leq 3$
 $\Rightarrow D_f = [1, 3]$ ✓

الف) $y = \frac{\sqrt{x^2-7x+6}}{\sqrt{x-10x+16}} \Rightarrow \begin{cases} x^2-7x+6 \geq 0 \rightarrow (x-1)(x-6) \geq 0 \rightarrow x \leq 1 \text{ or } x \geq 6 \\ x-10x+16 \geq 0 \rightarrow x \leq \frac{16}{9} \end{cases}$
 $(I) \cap (II) \rightarrow D_f = (-\infty, 1]$ ✓

ب) $y = \sqrt{\frac{x^2-7x+6}{x^2-10x+16}} \rightarrow \frac{(x-1)(x-6)}{(x-2)(x-8)} \geq 0 \rightarrow D_f = (-\infty, 1] \cup (2, 6] \cup (8, +\infty)$ ✓

الف) $y = \frac{\sqrt{2x^2-5x+3}}{\sqrt{3x^2-7x+4}}$ $2x^2-5x+3 \geq 0 \rightarrow (2x-3)(x-1) \geq 0 \rightarrow x \leq \frac{3}{2} \text{ or } x \geq 1$
 $3x^2-7x+4 \geq 0 \rightarrow (3x-4)(x-1) \geq 0 \rightarrow x \leq \frac{4}{3} \text{ or } x \geq 1$
 $(I) \cap (II) \rightarrow D_f = (-\infty, 1] \cup [\frac{4}{3}, \frac{3}{2}] \cup [\frac{3}{2}, +\infty)$ ✓

ب) $y = \sqrt{\frac{2x^2-5x+3}{3x^2-7x+4}} \rightarrow \frac{(2x-3)(x-1)}{(3x-4)(x-1)} \geq 0 \rightarrow D_f = (-\infty, 1] \cup (\frac{4}{3}, \frac{3}{2}] \cup [\frac{3}{2}, +\infty)$ ✓