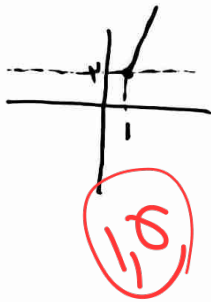


14, 20

الف) $n^2 - n = 2$ $n \in \mathbb{Z}^{-}$ $\rightarrow m=3, n=2$ ✓

ب) $m=2 \rightarrow n-1=1$ $\rightarrow a=-1$ $a+2=1$ $\rightarrow n=2$ ✓

2



$$f(n) = \begin{cases} 2n-1 & ; n \geq 1 \\ n+a & ; n < 1 \end{cases}$$

$$a \leq (f(1)-1)$$

$$\rightarrow a \leq 2$$

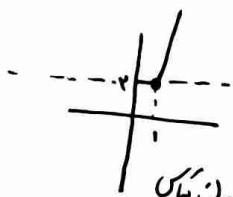
شیب + ✓

$$1+a \leq 2$$

$$a \leq 1$$

1

2



$$f(n) = \begin{cases} 2n-1 & ; n \geq 1 \\ n+a-1 & ; n \leq 0 \end{cases}$$

$$a > 0 \text{ شیب + } \checkmark$$

$$a-1 < 2 \quad a < 3$$

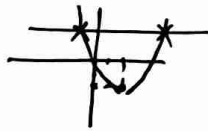
$$0 < a < 3$$

3

الف) $y = x^3 + 2$ $y_1 = y_2 \rightarrow x_1^3 + 2 = x_2^3 + 2$ $x_1 = x_2$ ✓
 $x = y^3 + 2 \rightarrow y = \sqrt[3]{x-2}$ ✓

2

ب) $x^2 - 5x + 2$ $\frac{b}{2a} = 2$ $(2, -2)$



(خط افقی دو بار قطع نمی کند)

4

الف) $y = \frac{2x+1}{x-2}$



یک به یک ✓

$$x = \frac{2y+1}{y-2} \rightarrow y = \frac{2x+1}{x-2}$$

5

ب) $y = \frac{5+2x}{x+2} = 2$

$$\rightarrow y=2$$



یک به یک نیست ✓

2

الف $y = \sqrt{x-3}$ $x \in [3, +\infty) \rightarrow y, y^2 \Rightarrow \sqrt{x-3} = \sqrt{y^2-3}$
 $x = y^2-3 \rightarrow x^2 = y^2-3 \rightarrow y = x^2+3$ ✓
 $n_1 = n_2 = n_3 = 3$ یک یک یک ✓
1/28

$\rightarrow x = y^2 - 3y + 3 + 1 - 1 \Rightarrow x = (y-1)^2 - 1 \Rightarrow y = \sqrt{x+1} + 1$
 $\therefore y = x^2 - 6x + 3; x \in [3, +\infty) \quad \frac{-b}{2a} = 1 \quad (1, -1)$ ✓
 $\frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{-3}}{2}$ یک یک ✓

$f(x) = x + \sqrt{x}$ $y = u^2 + u$
 $\hookrightarrow u^2 + u - y = 0 \rightarrow \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-1 \pm \sqrt{1+4y}}{2}$
 $x = \left(\frac{-1 + \sqrt{1+4y}}{2} \right)^2$
 $\Rightarrow x = \frac{1}{4} + y - \frac{1}{4}\sqrt{4+4y}$
 $\Rightarrow f^{-1}(x) = x + \frac{1}{4} - \frac{1}{4}\sqrt{4+4x} = ax + b - \sqrt{x-c} \rightarrow 1x + \frac{1}{4} - \sqrt{x - (-\frac{1}{4})}$
 $\left. \begin{matrix} a=1 \\ b=\frac{1}{4} \\ c=-\frac{1}{4} \end{matrix} \right\} ax+by+cz = 1$ ✓
2

$y = \frac{x}{\sqrt{x^2-2}}$ $D_f = (-\infty, -\sqrt{2}) \cup (\sqrt{2}, +\infty) \rightarrow x \neq 0$
 $a = \frac{y}{\sqrt{y^2-2}} \xrightarrow{\text{معکوس}} y = \frac{\sqrt{2}}{\sqrt{a^2-1}}$
 $y, y^2 \Rightarrow \left(\frac{x_1}{\sqrt{x_1^2-2}} = \frac{x_2}{\sqrt{x_2^2-2}} \right)^2$
 $\frac{x_1^2}{x_1^2-2} = \frac{x_2^2}{x_2^2-2} \Rightarrow x_1^2(x_2^2-2) = x_2^2(x_1^2-2)$
 $x_1^2 x_2^2 - 2x_1^2 = x_1^2 x_2^2 - 2x_2^2 \Rightarrow -2x_1^2 = -2x_2^2 \Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2$
 $x_1 = x_2$ ✓
 یک به یک است ← زیرا یک ی می توانند در دو قدرتی داشته باشند
 غلطی! 5

$g^{-1}(-\frac{r}{8}) = a$ $\frac{r}{8}$
 $\hookrightarrow g(a) = -\frac{r}{8} \Rightarrow \sqrt{a} - 1 = -\frac{r}{8}$
 $\boxed{a = \frac{9}{r^2}}$
 $\frac{b}{1+|b|} = -\frac{r}{8} \Rightarrow \frac{b}{1-b} = -\frac{r}{8} \Rightarrow \boxed{b = -\frac{r}{r}}$
 $a+b = -\frac{r}{r} + \frac{9}{r^2} = \boxed{-\frac{r^2}{r^2}}$ ✓
 $b < 0$ ✓

$g^{-1}(12) = a$
 $g(a) = 12 = f(a) + \sqrt{f(a)}$
 $\sqrt{f(a)} = t$
 $t^2 + t - 12 = 0 \rightarrow t = 3$ ✓
 $\sqrt{f(a)} = 3 \Rightarrow f(a) = 9 \Rightarrow f^{-1}(9) = a \Rightarrow \boxed{a=2}$
 $g^{-1}(12) = a \Rightarrow \boxed{g^{-1}(12) = 2}$ ✓
2