

الف) $f = \{(-1, 4), (m, 2), (1, n^2 - n), (n, 5), (3, 2)\}$

$n^2 - n = 2 \rightarrow n^2 - n - 2 = 0 \rightarrow (n-2)(n+1) = 0 \Rightarrow \begin{cases} n = -1 \times \\ n = 2 \checkmark \end{cases}$ و $m = 3$

ب) $f = \{(-1, 1), (1, 2), (2, 3), (a, m-1), (a+1, n), (m, 3)\}$

$(2, 3) \leftrightarrow (m, 3) \Rightarrow m = 2$ $m-1 = 1 \rightarrow (a, 1) \leftrightarrow (-1, 1) \Rightarrow a = -1$
 $a+1 = 2 \rightarrow (1, n) \leftrightarrow (1, 2) \Rightarrow n = 2$

$f(x) = \begin{cases} 3x-1; & x \geq 1 \\ x+a; & x < 1 \end{cases}$ $x=1 \rightarrow f(1) = 3 \cdot 1 = 2$
 $\Rightarrow 1+a < 2 \Rightarrow a < 1$

$\Rightarrow a = (-\infty, 1)$

$f(x) = \begin{cases} 3x-1; & x \geq 1 \rightarrow f(1) = 3 \cdot 1 - 1 = 2 \\ ax+a-1; & x \leq 0 \rightarrow f(0) = a \cdot 0 + a - 1 = a-1 \end{cases} \Rightarrow a-1 < 2 \rightarrow a < 3$

$a \neq 0 \rightarrow$ شیب خط صفر باشد

$a = (-\infty, 3) - \{0\}$

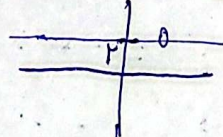
الف) $y = x^3 + 2$ $\checkmark$ $y = x^3 + 2 \rightarrow x^3 = y - 2 \rightarrow x = \sqrt[3]{y-2}$
 $\Rightarrow f^{-1}(x) = \sqrt[3]{x-2}$

ب) $y = x^2 - 2x + 2$ $\times$

چون تابع سهمی است پس نمودار آن یک خط افقی را در ۲ نقطه قطع می کند پس یک به یک نیست

الف) $y = \frac{x+1}{x-2}$ $f(x_1) = f(x_2) \rightarrow \frac{x_1+1}{x_1-2} = \frac{x_2+1}{x_2-2} \rightarrow -2x_1 + x_2 = -2x_2 + x_1 \rightarrow x_1 = x_2 \checkmark$

$y = \frac{x+1}{x-2} \rightarrow y(x-2) = x+1 \rightarrow x(y-3) = y+1 \rightarrow x = \frac{y+1}{y-3} \rightarrow f^{-1}(x) = \frac{x+1}{x-3}$

ب) $y = \frac{x^2+2x}{x+2} \rightarrow \frac{x(x+2)}{x+2} = 2 \rightarrow$  چون یک به یک نیست پس معکوس ندارد

الف) $y = \sqrt{x-3} \rightarrow$ یک یک $\rightarrow x = \sqrt{y-3} \rightarrow x^2 = y-3 \rightarrow y = x^2+3 \rightarrow f^{-1}(x) = x^2+3, x \geq 3$

ب) $y = x^2 - (x+3) \Rightarrow \frac{-b}{2a} = \frac{+f}{2} = 2 \Rightarrow$ یک یک $\rightarrow$

$y = x^2 - (x+3) = (x-2)^2 - 1 \rightarrow y+1 = (x-2)^2 \rightarrow x = \sqrt{y+1} + 2 \rightarrow f^{-1}(x) = \sqrt{x+1} + 2, x \geq -1$

$f(x) = x + \sqrt{x} \quad f(f^{-1}(x)) = x \rightarrow (ax+b - \sqrt{x-c}) + \sqrt{ax+b - \sqrt{x-c}}$

$ax+b - \sqrt{x-c} + \sqrt{ax+b - \sqrt{x-c}} = x \quad y = x + \sqrt{x} \xrightarrow{t=\sqrt{x}} y = t^2 + t \rightarrow t^2 + t - y = 0$

$t = \frac{-1 \pm \sqrt{1+4y}}{2} \rightarrow x = t^2 \rightarrow \left(\frac{-1 \pm \sqrt{1+4y}}{2}\right)^2 \rightarrow x = y + \frac{1}{4} - \frac{1}{4} \sqrt{1+4y} \rightarrow f^{-1}(y) = y + \frac{1}{4} - \frac{1}{4} \sqrt{1+4y}$

$f^{-1}(x) = 1 \times x + \frac{1}{4} - \frac{1}{4} \sqrt{1+4x} = x + \frac{1}{4} - \sqrt{x + \frac{1}{4}} \iff a=1, b=\frac{1}{4}, c=-\frac{1}{4}$

$y = \frac{x}{\sqrt{x^2-2}} \rightarrow f(x_1) = f(x_2) \rightarrow \frac{x_1^2}{x_1^2-2} = \frac{x_2^2}{x_2^2-2} \rightarrow x_1^2 x_2^2 - 2x_1^2 = x_1^2 x_2^2 - 2x_2^2$

$\rightarrow x_1^2 = x_2^2 \rightarrow x_1 = \pm x_2$ $\xrightarrow{\text{یک یکنوی}} x_1 = x_2$

$y = \frac{x}{\sqrt{x^2-2}} \rightarrow x = \frac{y}{\sqrt{y^2-2}} \rightarrow x^2 = \frac{y^2}{y^2-2} \rightarrow y^2 x^2 - 2x^2 = y^2 \rightarrow y^2 x^2 - y^2 = 2x^2$

$\rightarrow y^2 = \frac{2x^2}{x^2-1} \rightarrow y = \pm \sqrt{\frac{2x^2}{x^2-1}} \rightarrow y = \frac{\sqrt{2}x}{\sqrt{x^2-1}}$

$f^{-1}(x) = \frac{x}{1+|x|}, g(x) = \sqrt{x-1}$

$f^{-1}(x) \Rightarrow y = \frac{x}{1+|x|}, x > 0 \rightarrow x = \frac{y}{1-y}$

$y = \frac{x}{1-x}, x < 0 \rightarrow x = \frac{y}{1+y}$

$\Rightarrow g^{-1}\left(-\frac{y}{\omega}\right) + f\left(-\frac{y}{\omega}\right) = \frac{9}{\omega} - \frac{y}{\omega} = \frac{9-y}{\omega}$

$g^{-1}(x) \Rightarrow y = \sqrt{x-1} \rightarrow x = (y+1)^2$

$g^{-1}(x) = (x+1)^2$

$g^{-1}\left(-\frac{y}{\omega}\right) = \left(-\frac{y}{\omega} + 1\right)^2 = \frac{9}{\omega^2}$

$f^{-1}(x) = \sqrt[3]{x-1} \rightarrow x = \sqrt[3]{y-1} \rightarrow x^3 = y-1 \rightarrow f(x) = x^3+1$

$g(x) = f(x) + \sqrt{f(x)} \rightarrow (x^3+1) + \sqrt{x^3+1} = 12 \Rightarrow (x^3+1) + \sqrt{x^3+1} = 12$

$t = \sqrt{x^3+1} \Rightarrow t^2 + t - 12 = 0 \Rightarrow (t-3)(t+4) = 0 \Rightarrow \begin{cases} t=3 \\ t=-4 \end{cases}$

$\rightarrow t=3 \Rightarrow t^2 = x^3+1 \rightarrow x^3 = 8 \Rightarrow x = 2 \Rightarrow g^{-1}(12) = 2$