

الف) $f = \{(x, x), (n, 0), (x, n-x), (m, x), (-1, x)\}$ است

$n-x \geq x \Rightarrow n \geq 1 \rightarrow f = \{(-1, 0), (-1, x), \dots\} \Rightarrow$ غلط
 $n \geq 0$

است $\Rightarrow n \leq x$

ب) $f = \{(-1, 1), (1, 2), (x, x), (a, n-1), (a+x, n), (m, x)\}$

$\Rightarrow n \geq 2 \Rightarrow f = \{(-1, 1), (1, 2), (x, x), (a, 1), (a+x, n)\}$

$\Rightarrow a \geq 1 \Rightarrow f = \{(-1, 1), (1, 2), (x, x), (1, n)\} \Rightarrow n \geq 2$

$f(x) = \begin{cases} x-1; x \geq 1 \\ x+a; x < 1 \end{cases}$

۱- باید یک به یک باشد $a \neq 0$
۲- بردارها باید عضو شمر باشند.

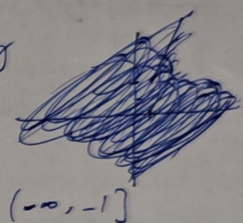
$y_1 = x-1; x \geq 1 \Rightarrow R_{y_1} = [x, +\infty)$

$y_2 = a+x-1; x < 1 \Rightarrow R_{y_2} = (-\infty, a+1]$

$[x, +\infty) \cap (-\infty, a+1] = \emptyset$

~~$x \geq a+1 \Rightarrow a \leq 1$~~

$\Rightarrow x \geq a+1 \Rightarrow a \leq 1$



$f(x) = \begin{cases} x-1; x \geq 1 \\ a+x-1; x < 0 \end{cases}$

$y_1 = x-1; x \geq 1 \Rightarrow R_{y_1} = [x, +\infty)$

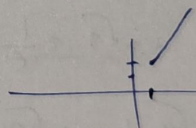
$y_2 = a+x-1; x < 0 \Rightarrow R_{y_2} = (-\infty, a-1]$

$(-\infty, a-1] \cap [x, +\infty) = \emptyset$

$a-1 < x \Rightarrow a < x$

$\Rightarrow 0 < a < x$

$a > 0$



$\Rightarrow x^3 + x = x^3 + x \Rightarrow x_1 = x_2$

الف) $y = x^3 + x \quad y_1 = y_2$

$x^3 = y - x \Rightarrow x = \sqrt[3]{y-x} \Rightarrow y = \sqrt[3]{x-x} \Rightarrow f^{-1}(3) = \sqrt[3]{3-3}$

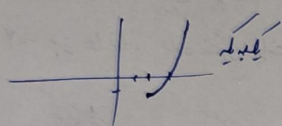
ب) $y = x^3 - 5x + x \Rightarrow$ سه به یک نیست

$a) y = \frac{x^2 + 1}{x - 2}$ بجای x از $y+1$ استفاده می‌کنیم ۱۵
 $x = y + 1$ $\Rightarrow y = \frac{(y+1)^2 + 1}{y+1 - 2} = \frac{(y+1)^2 + 1}{y-1}$

$b) y = \frac{x^2 + 2x + 2}{x + 1}$ بجای x از $y-1$ استفاده می‌کنیم

$a) y = \sqrt{x-3}$ بجای y از $x^2 + 3$ استفاده می‌کنیم ۱۶
 $x = y^2 + 3$
 $\Rightarrow f'(x) = 2x + 3$
 $R_f = D_f^{-1} = [0, +\infty)$

$b) y = x^2 - 5x + 4, x \in [1, +\infty)$



$y = x^2 - 5x + 4 \Rightarrow y = (x-2)^2 - 1$

$\Rightarrow x = \pm \sqrt{y+1} + 2 \Rightarrow y = 2 + \sqrt{x+1} = f^{-1}(x) = \sqrt{x+1} + 2$

$D_f = D_f^{-1} = [1, +\infty)$

$R_f = D_f^{-1} = [-1, +\infty)$

$y = x + \sqrt{x} \quad y_1 = y(x_1) = x_1 + \sqrt{x_1} = x_2 + \sqrt{x_2} = y_2 \Rightarrow (x_2 - x_1)(\sqrt{x_2} - \sqrt{x_1}) = 0$ ۱۷

$\Rightarrow (\sqrt{x_2} - \sqrt{x_1})(\sqrt{x_2} + \sqrt{x_1} + 1) = 0 \quad \Rightarrow \sqrt{x_2} - \sqrt{x_1} = 0 \Rightarrow \sqrt{x_2} = \sqrt{x_1}$

$\Rightarrow x_2 = x_1 \Rightarrow$

$y = x + \sqrt{x} \quad \Rightarrow y = x + \sqrt{x} + \frac{1}{x} - \frac{1}{x} = (\sqrt{x} + \frac{1}{x})^2 - \frac{1}{x} \Rightarrow y + \frac{1}{x} = (\sqrt{x} + \frac{1}{x})^2$

$\Rightarrow \sqrt{x} + \frac{1}{x} = \pm \sqrt{y + \frac{1}{x}} \Rightarrow \sqrt{x} = \pm \sqrt{y + \frac{1}{x}} - \frac{1}{x} \Rightarrow \sqrt{x} = \sqrt{y + \frac{1}{x}} - \frac{1}{x}$
 $\Rightarrow y = x + \frac{1}{x} - \sqrt{x + \frac{1}{x}}$

$\Rightarrow f^{-1}(x) = x + \frac{1}{x} - \sqrt{x + \frac{1}{x}} \quad \Rightarrow \begin{cases} a = 1 \\ b = \frac{1}{x} \\ c = -\frac{1}{x} \end{cases} \Rightarrow a + b + c = 1$
 $f(x) = ax + b - \sqrt{x + c}$

$$y = \frac{x}{\sqrt{x^2-1}}$$

$$y_1 = y_2 \Rightarrow \frac{x_1}{\sqrt{x_1^2-1}} = \frac{x_2}{\sqrt{x_2^2-1}} \Rightarrow \frac{x_1^2}{x_1^2-1} = \frac{x_2^2}{x_2^2-1}$$

$$\Rightarrow \frac{x_1^2}{-1} = \frac{x_2^2}{-1} \Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2$$

به خاطر اینکه به سوال بر میگردیم و جواب نه بدست می آید، یکی از جواب ها غلط است

$$\Rightarrow x_1 = x_2$$

$$y = \frac{x}{\sqrt{x^2-1}} \Rightarrow y^2 = \frac{x^2}{x^2-1} \Rightarrow x^2 = \frac{y^2}{y^2-1} \Rightarrow x = \pm \sqrt{\frac{y^2}{y^2-1}} \Rightarrow x = \frac{y}{\sqrt{y^2-1}} \Rightarrow y = \frac{x}{\sqrt{x^2-1}}$$

میشه معادله رو برعکس کرد $\Rightarrow f^{-1}(x) = \frac{x}{\sqrt{x^2-1}}$

$$g(x) = \sqrt{x-1} \Rightarrow g^{-1}(\sqrt{x-1}) = x \quad \sqrt{x-1} = \frac{-x}{2} \Rightarrow \sqrt{x} = \frac{x}{2} \Rightarrow x = \frac{4}{x}$$

$$f^{-1}(x) = \frac{x}{1+|x|} \Rightarrow f\left(\frac{x}{1+|x|}\right) = x$$

$$\frac{x}{1+|x|} = -\frac{x}{2} \Rightarrow \frac{x}{2+2|x|} = -\frac{x}{2} \Rightarrow x = 2x-2 \Rightarrow x = -2$$

$$\Rightarrow f\left(-\frac{x}{2}\right) + g\left(-\frac{x}{2}\right) = \frac{4}{2\omega} - \frac{x}{2} = \frac{2\sqrt{1-\omega^2}}{\sqrt{\omega}} = -\frac{x}{2} \Rightarrow f\left(-\frac{x}{2}\right) = -\frac{x}{2}$$

$$g(x) = f(x) + \sqrt{f(x)} \Rightarrow g^{-1}(f(x) + \sqrt{f(x)}) = x$$

$$f^{-1}(x) = \sqrt{x-1} \Rightarrow f(\sqrt{x-1}) = x \Rightarrow f(x) = x^2+1$$

$t \Rightarrow t^2 = x-1 \Rightarrow x = t^2+1$

$$\Rightarrow g^{-1}(\sqrt{x^2+1} + x^2+1) = g^{-1}(12) \xrightarrow{g \circ g^{-1}} \sqrt{x^2+1} + x^2+1 = 12$$

$$\Rightarrow t^2+t = 12 \Rightarrow t^2+t-12=0 \Rightarrow (t+4)(t-3)=0 \Rightarrow t = -4 \Rightarrow \sqrt{x^2+1} = -4 \Rightarrow x^2+1 = 16 \Rightarrow x^2 = 15 \Rightarrow x = \pm\sqrt{15}$$

$$\Rightarrow g^{-1}(12) = x$$