





$$R_f = [0, +\infty)$$

(الف)

$$y = \sqrt{x-2} \rightarrow x = \sqrt{y-2}^2 + 2$$

$$\rightarrow x^2 = y-2 \rightarrow y = x^2 + 2, R_f = [0, +\infty)$$

(۲)

$$y = x^2 - 2x + 3, x \in [2, +\infty) \rightarrow x = y^2 - 2y + 3 \rightarrow x = (y-1)^2 - 1 \rightarrow \sqrt{x+1} = y-1$$

$$\rightarrow y = \sqrt{x+1} + 1, x \in (-1, +\infty)$$

$$(x_1, y) \xrightarrow{f} x_1 = x^2 \xrightarrow{f^{-1}} \frac{x_1}{x^2} = \frac{x^2}{x^2-2} \xrightarrow{f^{-1}} \frac{x^2}{x^2-2} = \frac{x^2}{x^2-2} \rightarrow x_1^2 = x^2 \rightarrow x_1 = \pm x$$

$$(x_2, y) \xrightarrow{f} \frac{x_2}{\sqrt{x^2-2}} = \frac{x^2}{\sqrt{x^2-2}} \xrightarrow{f^{-1}} \frac{x^2}{x^2-2} = \frac{x^2}{x^2-2} \rightarrow x_2 = x^2 \rightarrow x_2 = \pm x^2$$

←  $x_1$  نمی تواند  $x^2$  مخالف علامت باشد چون علامت چپ تغییر نمی کند پس  $x_1 = x$  است

$$\xrightarrow{f^{-1}} x^2 = \frac{y^2+2}{y^2-2} \rightarrow y^2 = \frac{2x^2}{x^2-1} \rightarrow y = \frac{\sqrt{2x}}{\sqrt{x^2-1}}$$

$$f^{-1}(n) = \sqrt{x-1} \rightarrow f(n) = x^2 + 1 \rightarrow g(n) = f(n) + \sqrt{f(n)}$$

$$= x^2 + 1 + \sqrt{x^2 + 1}$$

$$g^{-1}(12) \rightarrow (12, x) \rightarrow g(x) = (x, 12) \rightarrow x^2 + 1 + \sqrt{x^2 + 1} = 12$$

$$\rightarrow x = 2$$

چون  $g(2) = 12$  پس  $f^{-1}(12) = 2$

$$f^{-1}(n) = \frac{x}{1+x} \rightarrow f(n) = \frac{y}{1+y} \rightarrow x = \frac{y}{1+y} \rightarrow -\frac{2}{0} = \frac{y}{1+y} \rightarrow y = -\frac{2}{1+y}$$

$$g(n) = \sqrt{n-1} \rightarrow n = \sqrt{y-1} \rightarrow y = (n+1)^2 \rightarrow y = \left(\frac{2}{0}\right)^2 = \frac{4}{0} \rightarrow y = -\frac{2}{1+y}$$

$$f\left(-\frac{2}{0}\right) + g^{-1}\left(-\frac{2}{0}\right) = \frac{9}{20} - \frac{2}{0} = \frac{20-4}{20} = -\frac{16}{20}$$

$$f(x) = x + \sqrt{x} \rightarrow f^{-1}(n) = y + \sqrt{y} \rightarrow x = y + \sqrt{y} \rightarrow x = \left(\sqrt{y} + \frac{1}{\sqrt{y}}\right)^2 - \frac{1}{y}$$

$$\rightarrow x + \frac{1}{x} = \left(\sqrt{y} + \frac{1}{\sqrt{y}}\right)^2 \Rightarrow \sqrt{x + \frac{1}{x}} = \sqrt{y} + \frac{1}{\sqrt{y}} \rightarrow \sqrt{y} = \sqrt{x + \frac{1}{x}} - \frac{1}{\sqrt{y}}$$

$$\Rightarrow y = x + \frac{1}{x} + \frac{1}{x} - \sqrt{x + \frac{1}{x}} \rightarrow a = 1, b = \frac{1}{x}, c = -\frac{1}{x}$$

$$a + 2b + c = 1 + \frac{2}{x} - \frac{1}{x} = 1 + \frac{1}{x}$$

★ پاسخ سوالات ۸ و ۱۰ جایسته اند.