

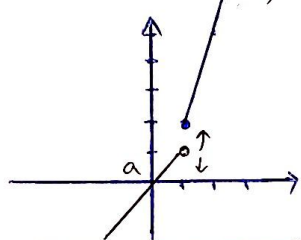
الف) $f = \{(3, 2), (n, 5), (3, n^2 - n), (m, 2), (-1, 2)\}$ $n^2 - n = 2 \rightarrow n^2 - n - 2 = 0 \rightarrow n = -1$ یا $n = 2$ ✓

$\rightarrow n = 2 \mid m = 3$

ب) $f = \{(-1, 1), (1, 2), (2, 3), (x, m-1), (a, n), (x, 2)\} \rightarrow m = 2 \rightarrow a = -1$

$\rightarrow n = 2$

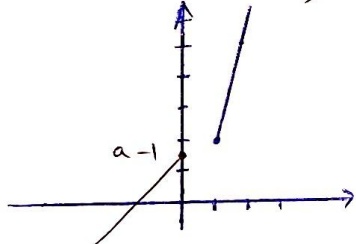
$f_{(n)} = \begin{cases} 3n - 1 & ; n \geq 1 \\ n + a & ; n < 1 \end{cases}$



$\rightarrow 1 + a \leq 2 \rightarrow a \leq 1$

$\rightarrow a \in (-\infty, 1]$

$f_{(n)} = \begin{cases} 3n - 1 & ; n \geq 1 \\ an + a - 1 & ; n < 1 \end{cases}$



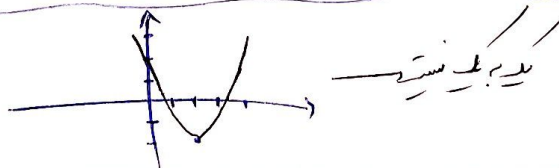
$\rightarrow \begin{cases} a > 0 \\ a - 1 < 2 \rightarrow a < 3 \end{cases} \rightarrow 0 < a < 3$

$\rightarrow a \in (0, 3)$

الف) $y = x^3 + 2$ $y = x^3 + 2 \rightarrow x^3 = y - 2 \rightarrow x = \sqrt[3]{y - 2}$

$\rightarrow y = \sqrt[3]{x - 2} \rightarrow f_{(x)}^{-1} = \sqrt[3]{x - 2}$

ب) $y = x^2 - 2x + 2$



الف) $y = \frac{3x+1}{x-2}$ $x = \frac{2y+1}{y-3}$

$\rightarrow f_{(x)}^{-1} = \frac{2x+1}{x-3}$

تابع معکوس هر دو معکوس است

ب) $y = \frac{4+2x}{x+1} \rightarrow y = \frac{2(x+2)}{x+1} = 2$



معکوس نیست

$$y = \sqrt{x-2} \quad \xrightarrow{\text{graph}} \quad y^2 = x-2 \rightarrow x = y^2+2 \rightarrow f_{(y)}^{-1} = y^2+2 \quad x \geq 0$$

$$y = x^2 - \varepsilon x + 2 \quad x \in [2, +\infty) \quad \xrightarrow{\text{graph}} \quad y = (x-2)^2 + 1 \rightarrow (x-2)^2 = y-1$$

$$\rightarrow (x-2) = \pm \sqrt{y+1} \rightarrow x = 2 \pm \sqrt{y+1} \rightarrow f_{(y)}^{-1} = 2 + \sqrt{y+1}$$

چون برای y معلوم، x را می‌توان به دو روش پیدا کرد (برای $x \geq 2$) $\leftarrow$ درست نمی‌باشد

$$f_{(x)} = x + \sqrt{x} \quad \xrightarrow{\sqrt{x}=t} \quad y = t^2 + t \rightarrow t^2 + t - y = 0 \xrightarrow{\Delta} t = \frac{-1 \pm \sqrt{1+4y}}{2}$$

$$\xrightarrow{\sqrt{x}} \sqrt{x} = \frac{-1 + \sqrt{1+4y}}{2} \rightarrow x = \frac{1+4y - 2\sqrt{1+4y} + 1}{4} = \frac{1}{4} + y - \frac{\sqrt{1+4y}}{2}$$

$$= \frac{1}{4} + y - \sqrt{\frac{1}{4} + y} \rightarrow y = x + \frac{1}{4} - \sqrt{\frac{1}{4} + x} \rightarrow \begin{cases} a=1 \\ b=\frac{1}{4} \\ c=-\frac{1}{2} \end{cases}$$

$$\rightarrow a + 4b + 4c = 1 + 1 - 1 = 1$$

$$y = \frac{x}{\sqrt{x^2-1}} \rightarrow f_{(x_1)} = f_{(x_2)} \rightarrow \frac{x_1}{\sqrt{x_1^2-1}} = \frac{x_2}{\sqrt{x_2^2-1}} \rightarrow \text{خرج هم علامت است}$$

$$\xrightarrow{\text{چون}} \frac{x_1^2}{x_1^2-1} = \frac{x_2^2}{x_2^2-1} \rightarrow x_1^2(x_2^2-1) = x_2^2(x_1^2-1) \rightarrow x_1^2 = x_2^2 \rightarrow x_1 = \pm x_2$$

$$\xrightarrow{\text{کلیه } f^{-1}} x = \frac{y}{\sqrt{y^2-1}} \rightarrow x^2 = \frac{y^2}{y^2-1} \rightarrow y^2 = \frac{x^2}{x^2-1} \rightarrow y = \pm \sqrt{\frac{x^2}{x^2-1}}$$

$$\rightarrow y = \sqrt{\frac{x^2}{x^2-1}} \rightarrow f_{(x)}^{-1} = \frac{\sqrt{x^2}}{\sqrt{x^2-1}} \quad \leftarrow \text{هم علامت است}$$

$$g_{(x)} = \sqrt{x}-1 \quad f_{(x)}^{-1} = \frac{x}{1+|x|} \rightarrow f_{(-\frac{1}{2})} + g_{(-\frac{1}{2})} = \frac{1}{1.5} - \frac{1}{2} = -\frac{1}{3}$$

$$\begin{cases} \frac{x}{1+|x|} = -\frac{1}{2} \xrightarrow{x < 0} \frac{x}{1-x} = -\frac{1}{2} \rightarrow x = -\frac{1}{3} \rightarrow f_{(-\frac{1}{2})} = -\frac{1}{3} \end{cases}$$

$$|\sqrt{x}-1| = -\frac{1}{2} \rightarrow \sqrt{x} = \frac{1}{2} \rightarrow x = \frac{1}{4} \rightarrow g_{(-\frac{1}{2})} = \frac{1}{4}$$

$$f_{(x)}^{-1} = \sqrt{x-1} \quad g_{(x)} = f_{(x)} + \sqrt{f_{(x)}} \quad g_{(12)}^{-1} = ?$$

$$\rightarrow g_x = \sqrt{f(x)} + f(x) = 12 \rightarrow f_x = 9$$

$$\rightarrow f_{(9)}^{-1} = x \rightarrow \sqrt{9-1} = x \rightarrow \boxed{x=2} \rightarrow g_x = 12 \rightarrow \boxed{g_{(12)}^{-1} = 2}$$