

نام و نام خانوادگی: شماره کلاس: شماره برگه: پاسخنامه تشریحی تکلیف شماره ۵۰

$$f = \{(3, 2), (n, 5), (3, n^2 - n), (m, 2), (-1, 4)\}$$

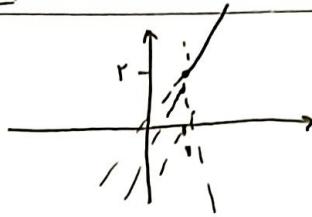
$$(3, 2) = (m, 2) \rightarrow m = 3 \quad (3, 2) \neq (3, n^2 - n) \rightarrow n^2 - n - 2 = -(n-2)(n+1) = 0$$

$$\rightarrow n = 2, -1 \rightarrow \text{تابع نیست} \rightarrow \boxed{m = 2}$$

$$f = \{(-1, 1), (1, 2), (2, 2), (a, m-1), (a+2, n), (m, 2)\} \rightarrow \boxed{m = 2} \rightarrow m-1 = 1$$

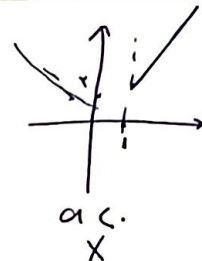
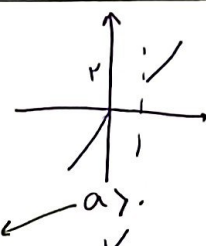
$$\rightarrow \boxed{a = -1} \rightarrow a+2 = 1 \rightarrow \boxed{n = 2}$$

$$f(x) = \begin{cases} 3x-1; & x \geq 1 \\ x+a; & x < 1 \end{cases}$$



$$\rightarrow x+a \leq 2 \xrightarrow{x \rightarrow 1} a \leq 1$$

$$f(x) = \begin{cases} 3x-1; & x \geq 1 \\ ax+a-1; & x < 1 \end{cases}$$

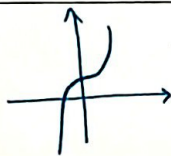


$$ax+a-1 < 2 \xrightarrow{x \rightarrow 1} a-1 < 2$$

$$\rightarrow \boxed{a < 3}$$

$$\rightarrow \boxed{a < 3} \rightarrow (0, 3)$$

$$y = x^3 + 2$$

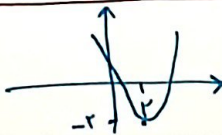


یک به یک

$$y = x^3 + 2 \rightarrow y - 2 = x^3 \rightarrow \sqrt[3]{y-2} = x$$

$$\rightarrow \boxed{f^{-1}(x) = \sqrt[3]{x-2}}$$

$$y = x^2 - 4x + 2$$



یک به یک نیست

$$y = \frac{3m+1}{m-2}$$

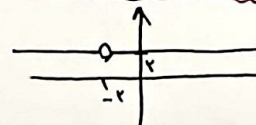
هرگز

یک به یک

$$\frac{3m+1}{m-2} = y \rightarrow -\frac{2y-1}{y-3} = m$$

$$\rightarrow \boxed{y = \frac{2m+1}{m-3}}$$

$$y = \frac{4+2m}{m+2} \rightarrow y = 2$$



یک به یک نیست

$$y = \sqrt{x-r} \rightarrow \text{graph} \rightarrow \text{domain}$$

$$y = \sqrt{x-r} \rightarrow y^2 = x-r \rightarrow y^2 + r = x \xrightarrow{\text{sub}} \boxed{x^2 + r, y} \rightarrow x \geq 0$$

$$y = x^2 - 5x + 2 \quad x \in [r, +\infty)$$

$$\xrightarrow{\text{sub}} y+1 = x^2 - 5x + 3 \rightarrow y+1 = (x-r)^2 \rightarrow \sqrt{y+1} = x-r \rightarrow \sqrt{y+1} + r = x$$

$$f(x) = x + \sqrt{x} \quad \sqrt{x} = t \rightarrow y = t + t^2 \rightarrow t^2 + t - y = 0$$

$$\Delta \rightarrow t = \frac{-1 \pm \sqrt{1+4y}}{2} \quad \text{then } \sqrt{x} = \frac{-1 + \sqrt{1+4y}}{2} \xrightarrow{\text{sub}} x = \left(\frac{-1 + \sqrt{1+4y}}{2} \right)^2$$

$$\rightarrow \frac{1}{r} + y - \frac{1}{r} \sqrt{4y+1} = \frac{1}{r} + y - \sqrt{y + \frac{1}{r}} \rightarrow y = \frac{1}{r} + x - \sqrt{x + \frac{1}{r}}$$

$$a+rb+rc \Rightarrow \frac{a}{b} = \frac{1}{r} \quad \frac{c}{b} = \frac{1}{r} \Rightarrow 1+1-1 = 1$$

$$y = \frac{x}{\sqrt{x^2-r}} \xrightarrow{\text{sub}} \frac{x_1}{\sqrt{x_1^2-r}} = \frac{x_r}{\sqrt{x_r^2-r}} \xrightarrow{\text{sub}} \frac{x_1^2}{x_1^2-r} = \frac{x_r^2}{x_r^2-r}$$

$$\rightarrow x_1^2 \cancel{x_r^2} - r x_1^2 = x_r^2 \cancel{x_1^2} - r x_r^2 \rightarrow x_1^2 = x_r^2 \rightarrow x_1 = \pm x_r$$

$$f(x) \neq f(-x) \rightarrow \text{not odd} \rightarrow \text{graph} \rightarrow y = \frac{x}{\sqrt{x^2-r}} \xrightarrow{\text{sub}} y^2 = \frac{x^2}{x^2-r} \rightarrow y^2(x^2-r) = x^2 \rightarrow y^2 x^2 - y^2 r = x^2 \rightarrow y^2 x^2 - x^2 = y^2 r \rightarrow x^2(y^2 - 1) = y^2 r$$

$$\rightarrow \frac{x^2}{y^2 - 1} = \frac{y^2 r}{y^2 - 1} \rightarrow \frac{x^2}{y^2 - 1} = y^2 \rightarrow x = \pm \frac{y \sqrt{r}}{\sqrt{y^2 - 1}} \rightarrow \boxed{y = \frac{x \sqrt{r}}{\sqrt{x^2 - r}}}$$

$$f^{-1}(x) = \frac{x}{1+|x|} \rightarrow f\left(-\frac{r}{a}\right) \rightarrow \frac{x}{1+|x|} = -\frac{r}{a} \rightarrow x = -\frac{r}{a} - r|x|$$

$$\xrightarrow{\text{sub}} \frac{x}{r} = -\frac{r}{a} \rightarrow x = -\frac{r}{a}$$

$$g(x) = \sqrt{x-1} \rightarrow \sqrt{x-1} = \frac{r}{a} \rightarrow \sqrt{x} = \frac{r}{a} + 1 \rightarrow x = \left(\frac{r}{a} + 1 \right)^2$$

$$f\left(-\frac{r}{a}\right) + g^{-1}\left(-\frac{r}{a}\right) = \frac{r}{a} + \frac{r}{a} \rightarrow \frac{2r}{a} = \frac{-r}{\sqrt{a}} \rightarrow \boxed{\frac{-r}{\sqrt{a}}}$$

$$f^{-1}(x) = \sqrt{x-1} \rightarrow y^2 = x-1 \rightarrow y^2 + 1 = x \xrightarrow{\text{sub}} \boxed{x^2 + 1, y}$$

$$g(x) = f(x) + \sqrt{f(x)} \rightarrow g^{-1}(11) \rightarrow f(x) + \sqrt{f(x)} = 11$$

$$\rightarrow f(x) = 9 \rightarrow x^2 + 1 = 9 \rightarrow \boxed{x = 2}$$