

$$f(n) = \begin{cases} n^3 - 1 & ; n > a \\ 1(n-1) & ; n \leq a \end{cases} \xrightarrow{n=a} a^3 - 1 \geq 1(a-1) \rightarrow a^3 - 1(a+1) \geq 0$$

$$\Rightarrow (a+1)(a-1)^2 \geq 0 \Rightarrow \frac{-1}{-1+1+1} = a \geq 1 \quad (2)$$

$$f(n) = 1n + k \text{ و } f^{-1}(1) = f \Rightarrow f(1) = 1 \rightarrow 1 + k = 1 \quad k = 0 \Rightarrow f(n) = 1n - 1$$

الف) $f(1) = 1 - 1 = 0$ ✓ (2)

ب) $f(f(n)) = f \circ f(n) = 1(1n - 1) - 1 = 1n - 2$ ✓

$$A(1a, a) \text{ و } f(n) = \frac{an}{n-1} \Rightarrow A(1a, a) \rightarrow f(a) = 1a$$

$$f(a) = \frac{a^2}{a-1} = 1a \rightarrow a^2 = 1a^2 - 1a \rightarrow a(a-1) = 0 \begin{cases} a=1 \rightarrow X \text{ دایره ندارد} \\ a=0 \rightarrow \checkmark \end{cases} \quad (2)$$

$$f = \{(1, \omega), (1, \nu), (1, 1), (1, 1)\}, g = \{(1, 1), (1, 1), (1, \omega)\}$$

الف) $f \circ f^{-1} = \{(1, 1), (\omega, \omega), (1, 1), (1, 1)\}$ ✓

ب) $f^{-1} \circ f = \{(1, 1), (1, 1), (1, 1), (1, 1)\}$ ✓ (2)

ج) $f \circ g^{-1} = \{(1, \omega), (\omega, 1)\}$ ✓

د) $g^{-1} \circ f = \{(1, 1), (1, 1), (1, 1)\}$ ✓

$$f = \{(1, 1), (1, 1), (1, 1)\} \text{ و } g = \{(1, 1), (1, 1), (1, 1)\}, h = \{(1, 1), (1, 1), (1, 1)\}$$

$$\frac{h}{f \circ g^{-1}} \rightarrow f \circ g^{-1} = \{(1, 1), (1, 1), (\omega, 1)\} \Rightarrow \frac{h}{f \circ g^{-1}} = \{(1, 1), (\omega, 1)\} \quad (1, 1)$$

$$\frac{h}{f \circ g^{-1}} = \{(1, 1), (1, 1)\}$$

$y = \frac{x^3+1}{x-1} \rightarrow \text{میانبردی} = 1 \Rightarrow \text{دارن پذیراست}$ $y = \frac{x^3+1}{x-1} \rightarrow x = \frac{y+1}{y-1} \rightarrow xy - 2x = y+1$ $\rightarrow y(x-2) = y+1 \rightarrow y = \frac{y+1}{x-2} \rightarrow f^{-1}(x) = \frac{x+1}{x-2}$	
$f(x) = x-1 - x-2 \rightarrow \begin{cases} 1 & x \geq 2 \\ x-1 & 1 \leq x \leq 2 \\ -1 & x \leq 1 \end{cases}$ $\Rightarrow f^{-1}(x) = \frac{x+1}{2} \quad -1 \leq x \leq 1$	
$f(x) = \begin{cases} x^2+1 & x \geq 1 \\ f(x-1) & x < 1 \end{cases}$ $f^{-1}(x) = \begin{cases} \sqrt{x-1} & x \geq 1 \\ \frac{x+1}{2} & x < 1 \end{cases}$	
$f(x) = x^2 - \frac{(x+1)^3}{x+3}, f^{-1}(x) = \frac{ax+b}{cx+d}$ $f(x) = \frac{x^3 + 3x^2 - x^3 - 3x^2 - 3x - 1}{x+3} = \frac{-3x-1}{x+3}$ $a=-3, b=-1, c=1, d=3$ $f^{-1}(-1) = \frac{3(-1)-1}{-1+3} = -2$	
$y = \frac{x}{x^2+1} \rightarrow x = \frac{y}{y^2+1} \rightarrow xy^2 + x = y \rightarrow xy^2 - y + x = 0$ $\Rightarrow y = \frac{1 \pm \sqrt{1-4x^2}}{2x}$ $f^{-1}(x) = \frac{1 \pm \sqrt{1-4x^2}}{2x}, x \in [-\frac{1}{2}, \frac{1}{2}], x \neq 0$	

چون x و y هم علامتین فقط + قاب قبول است