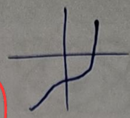
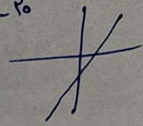
رادین ملکیان یازدهم A

19,75

۱)

$$f(x)=\begin{cases} x^3-4 & ; x>a \\ 12x-20 & ; x\le a \end{cases}$$

$y_1 = x^3-4$  $y_2 = 12x-20$ 

2

$x^3-4 \ge 12x-20 \Rightarrow x^3-4x-8x+16 \ge 0 \Rightarrow x(x^2-4)-8(x-2)\ge 0$

$\Rightarrow (x-2)(x^2+2x-8)\ge 0 \Rightarrow (x-2)^2(x+4)\ge 0 \Rightarrow$

	-4		2	
-	0	+	0	+

$\rightarrow [-4, +\infty)$ ✓

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۲)

$f^{-1}(2)=4 \Rightarrow f(4)=2 \qquad f(x)=3x+k \underset{x=4}{\Rightarrow} f(4)=12+k \Rightarrow k=-10$

$\Rightarrow f(x)=3x-10$

$f(7)=21-10=11$ ✓

2

$f(f(x)) = f(3x-10) = \boxed{} \Rightarrow f(f(x)) = 3(3x-10)-10 = 9x-30-10$

$\Rightarrow 9x-40$ ✓

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۳)

$f(a)=\dfrac{a^2}{a-1}=2a \Rightarrow a^2-2a=0 \Rightarrow a(a-2)=0 \Rightarrow a=0$ ×

$\boxed{a=2}$ ✓

2

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۴)

الف) $f\circ f^{-1} = \{(5,5),(7,7),(2,2),(6,6)\}$ ✓ $\quad 5\to 3\to 5,\ 7\to 4\to 7,\ 2\to 7\to 2,\ 6\to 9\to 6$

ب) $f^{-1}\circ f = \{(3,3),(4,4),(7,7),(9,9)\}$ ✓ $\quad 3\to 5\to 3,\ 4\to 7\to 4,\ 7\to 2\to 7,\ 9\to 6\to 9$

2

ج) $f\circ g^{-1} = \{(2,5),(5,6)\}$ ✓ $\quad 2\to 3\to 5,\ 6\to 8\to \times,\ 5\to 9\to 6$

د) $g^{-1}\circ f = \{(3,9),(7,3),(9,8)\}$ ✓ $\quad 3\to 5\to 9,\ 4\to 7\to \times,\ 7\to 2\to 3,\ 9\to 6\to 8$

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۵)

$f\circ g^{-1} = \{(1,4),(3,8),(5,0)\}$

$1\to 2\to 4$

$3\to 4\to 8$

$5\to 6\to 0$

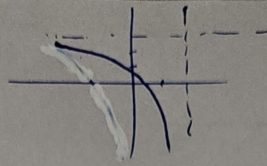
$\dfrac{h}{f\circ g^{-1}} = \{(1,\tfrac{2}{4}),(3,\tfrac{4}{8}),\cancel{(5,\tfrac{4}{0})}\}$

2

$\dfrac{h}{f\circ g^{-1}} = \{(1,\tfrac{1}{2}),(3,\tfrac{1}{2})\}$ ✓

$y = \dfrac{3x+1}{x-2}$ $\begin{cases} \text{مجانب افقی} = \dfrac{a}{c} = 3 \\ \text{مجانب قائم} = 2 \end{cases}$

$x = 0 \Rightarrow y = -\dfrac{1}{2}$

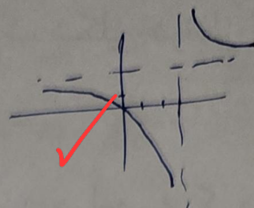


یک به یک

$f^{-1}(x) = \dfrac{2x+1}{x-3}$ → مجانب افقی $= \dfrac{a}{c} = 2$

مجانب قائم $= 3$

$x = 0 \Rightarrow f^{-1}(0) = -\dfrac{1}{3}$



۲

۷

$x = 1 \Rightarrow f(1) = -2$
$x = 3 \Rightarrow f(x) = 2$

$f(x) = |x-1| - |x-3|$

	1		3	
-2 نیست		$2x-4$ یک به یک		$x-1-x+3$ یک به یک نیست

$[1,3] \to a = 1, b = 3$

$\varnothing$

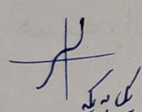
۲

$f^{-1}(x) = \dfrac{x+4}{2} = \dfrac{1}{2}x + 2 : -2 \le x \le 2 \Rightarrow R_f = D_{f^{-1}} = [-2, 2]$

۸

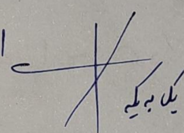
$f(x) = \begin{cases} x^3+4 & ; x \ge 1 \Rightarrow x = 1 \Rightarrow y = 5 \\ 4x-1 & ; x \le 0 \quad x = 0 \Rightarrow y = -1 \end{cases}$

$y_1 = x^3 + 4$



یک به یک

$y_2 = 4x - 1$



یک به یک

$\left.\begin{array}{l} R_{y_1} = [5, +\infty) \\ R_{y_2} = (-\infty, -1] \end{array}\right\} \cap \Rightarrow R_{g_1} \cap R_{g_2} = \varnothing$

۲

$f^{-1}(x) = \begin{cases} \sqrt[3]{x-4} & ; x \ge 5 \to R_{y_1} = [5, +\infty) = D_{g^{-1}} \\ \dfrac{1}{4}x + \dfrac{1}{4} & ; x \le -1 \to R_{y_2} = D_{g_2^{-1}} = (-\infty, -1] \end{cases}$

۹

$f(x) = x^2 - \dfrac{x^3+3x^2+3x+1}{x+3} = \dfrac{x^3+3x^2-x^3-3x^2-3x-1}{x+3} \Rightarrow f(x) = \dfrac{-3x-1}{x+3}$

$\Rightarrow f^{-1}(x) = \dfrac{-3x-1}{x+3} = \dfrac{-6x-2}{2x+6} = \dfrac{ax+b}{2x+d} \Rightarrow \begin{cases} a = -6 \\ b = -2 \\ d = 6 \end{cases}$

$\xrightarrow{x=-2} f^{-1}(-2) = \dfrac{6-1}{-2+3} = 5$

۲

$$f(x)=\frac{x}{x^2+1}$$

$$\left.\begin{array}{l} y=\dfrac{x_1}{x_1^2+1} \\ y=\dfrac{x_2}{x_2^2+1}\end{array}\right\} \quad \frac{x_1}{x_1^2+1}=\frac{x_2}{x_2^2+1} \Rightarrow x_1x_2^2+x_1=x_1^2x_2+x_2$$

$$\Rightarrow x_1x_2^2-x_1^2x_2+x_1-x_2=0$$

$$\Rightarrow x_1x_2(x_2-x_1)-(x_2-x_1)=0 \Rightarrow (x_1x_2-1)(x_2-x_1)=0$$

$$\Rightarrow \begin{cases} x_1=x_2 \\ x_1x_2=1 \end{cases}$$ ⇐ یک به یک نیست — به ازای مقدار معکوس یک جواب می دهد

چون x و y هم علامتند فقط $+$ قابل قبول است

$$y=\frac{x}{x^2+1} \Rightarrow yx^2+y=x \Rightarrow yx^2-x+y=0$$

$$\Rightarrow x=\frac{1\pm\sqrt{1-4y^2}}{2y} \Rightarrow y=\frac{1\pm\sqrt{1-4x^2}}{2x} \Rightarrow f^{-1}(x)=\frac{1\pm\sqrt{1-4x^2}}{2x}$$

$$f(2)=\frac{2}{5} \Rightarrow f^{-1}\left(\frac{2}{5}\right)=2 \Rightarrow f^{-1}(x)=\frac{1+\sqrt{1-4x^2}}{2x} \checkmark \qquad |x|\leq \frac{1}{2}$$

$$f^{-1}(x)=\frac{1-\sqrt{1-4x^2}}{2x} \xRightarrow{x=\frac{2}{5}} f^{-1}\left(\frac{2}{5}\right)=\frac{1-\sqrt{1-\frac{16}{25}}}{\frac{4}{5}}=\frac{\frac{2}{5}}{\frac{4}{5}}=\frac{1}{2}\neq 2$$

۱٫۷۵