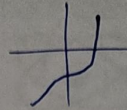


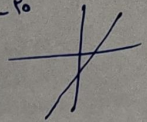
Algorithme CUBO

$$f(x) = \begin{cases} x^2 - 4 & ; x > a \\ 12x - 20 & ; x \leq a \end{cases}$$

$$y_1 = x^2 - 4$$



$$y_2 = 12x - 20$$



$$x^2 - 4 \geq 12x - 20 \Rightarrow x^2 - 12x + 16 \geq 0 \Rightarrow x(x - 4) - 1(x - 2) \geq 0$$

$$\Rightarrow (x - 2)(x^2 - 12x + 16) \geq 0 \Rightarrow (x - 2)^2(x + 4) \geq 0 \Rightarrow \frac{-4}{-6+6+4} \rightarrow [-4, +\infty)$$

$$f^{-1}(1) = 5 \Rightarrow f(5) = 1 \quad f(x) = 12x - 20 \Rightarrow f(5) = 12 \cdot 5 - 20 = 40 \Rightarrow 40 = 1 \Rightarrow 12x - 20 = 1 \Rightarrow 12x = 21 \Rightarrow x = \frac{7}{4}$$

$$f(5) = 12 \cdot 5 - 20 = 40$$

$$f(f(x)) = f(12x - 20) = \boxed{} \Rightarrow f(f(2)) = 12(12 \cdot 2 - 20) - 20 = 96 - 20 - 20 = 56$$

$$f(a) = \frac{a^2}{a-1} = 2a \Rightarrow a^2 - 2a = 0 \Rightarrow a(a-2) = 0 \Rightarrow a = 0 \text{ or } a = 2$$

$$a) f \circ f^{-1} = \{(2, 2), (5, 5), (7, 7), (9, 9)\}$$

$$\begin{array}{l} 2 \rightarrow 2 \rightarrow 2 \\ 5 \rightarrow 5 \rightarrow 5 \\ 7 \rightarrow 7 \rightarrow 7 \end{array}$$

$$b) f^{-1} \circ f = \{(2, 2), (5, 5), (7, 7), (9, 9)\}$$

$$\begin{array}{l} 2 \rightarrow 2 \rightarrow 2 \\ 5 \rightarrow 5 \rightarrow 5 \\ 7 \rightarrow 7 \rightarrow 7 \end{array}$$

$$c) f \circ g^{-1} = \{(2, 2), (2, 5)\}$$

$$\begin{array}{l} 2 \rightarrow 2 \rightarrow 2 \\ 5 \rightarrow 5 \rightarrow 5 \end{array}$$

$$d) g^{-1} \circ f = \{(2, 2), (5, 5), (9, 9)\}$$

$$\begin{array}{l} 2 \rightarrow 2 \rightarrow 2 \\ 5 \rightarrow 5 \rightarrow 5 \\ 9 \rightarrow 9 \rightarrow 9 \end{array}$$

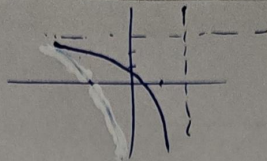
$$f \circ g^{-1} = \{(1, 1), (2, 2), (3, 3)\}$$

$$\begin{array}{l} 1 \rightarrow 1 \rightarrow 1 \\ 2 \rightarrow 2 \rightarrow 2 \\ 3 \rightarrow 3 \rightarrow 3 \end{array}$$

$$\frac{h}{f \circ g^{-1}} = \{(1, \frac{1}{2}), (2, \frac{1}{4}), (\cancel{3, \frac{1}{6}})\}$$

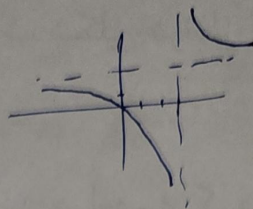
$$\frac{h}{f \circ g^{-1}} = \{(1, \frac{1}{2}), (2, \frac{1}{4})\}$$

$$y = \frac{x^2 + 1}{x - 2} \begin{cases} \text{مجاوب افق} = \frac{a}{c} = 2 \\ \text{مجاوب قائم} = 2 \end{cases}$$



→ مجاوب

$$f^{-1}(x) = \frac{x^2 + 1}{x - 3} \rightarrow \begin{cases} \text{مجاوب افق} = \frac{a}{c} = 2 \\ \text{مجاوب قائم} = 3 \end{cases}$$

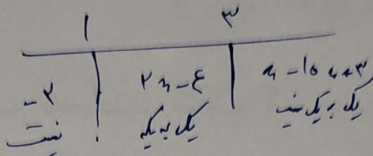


$$x = 0 \Rightarrow f^{-1}(1) = -\frac{1}{2}$$

$$a = 1 \Rightarrow f(a) = 2 \\ b = 3 \Rightarrow f(b) = 2$$

$$[1, 3] \rightarrow a = 1, b = 3$$

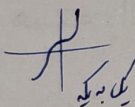
$$f(x) = |x - 1| - |x - 3|$$



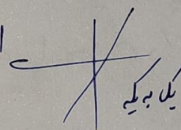
$$f^{-1}(x) = \frac{x + 5}{4} = \frac{1}{4}x + \frac{5}{4} : -2 \leq x \leq 2 \Rightarrow R_f = R_{f^{-1}} = [-2, 2]$$

$$f(x) = \begin{cases} x^2 + 1 & ; x \geq 1 \Rightarrow x = 1 \Rightarrow y = 2 \\ x - 1 & ; x \leq 0 \Rightarrow x = 0 \Rightarrow y = -1 \end{cases}$$

$$y_1 = x^2 + 1$$



$$y_2 = x - 1$$



$$\begin{cases} R_{y_1} = [0, +\infty) \\ R_{y_2} = (-\infty, 1] \end{cases} \Rightarrow R_{y_1} \cap R_{y_2} = \emptyset \checkmark$$

$$f^{-1}(x) = \begin{cases} \sqrt{x - 1} & ; x \geq 1 \rightarrow R_{y_1} = [0, +\infty) = R_{y_1}^{-1} \\ \frac{1}{x} x + \frac{1}{x} & ; x \leq -1 \rightarrow R_{y_2} = R_{y_2}^{-1} = (-\infty, -1] \end{cases}$$

$$f(x) = x^2 - \frac{x^3 + 3x^2 + 3x + 1}{x + 3} = \frac{x^3 + 3x^2 - x^3 - 3x^2 - 3x - 1}{x + 3} \Rightarrow f(x) = \frac{-3x - 1}{x + 3}$$

$$\Rightarrow f^{-1}(x) = \frac{-3x - 1}{x + 3} = \frac{-3x - 1}{x + 3} = \frac{ax + b}{cx + d} \Rightarrow \begin{cases} a = -3 \\ b = -1 \\ c = 1 \\ d = 3 \end{cases}$$

$$x = -2 \Rightarrow f^{-1}(-2) = \frac{-9 - 1}{-2 + 3} = 0$$

$$f(x) = \frac{x}{x^2+1}$$

$$y = \frac{x_1}{x_1^2+1}$$

$$y = \frac{x_2}{x_2^2+1}$$

$$\left\{ \begin{array}{l} \frac{x_1}{x_1^2+1} = \frac{x_2}{x_2^2+1} \Rightarrow x_1 x_2^2 + x_1 = x_1^2 x_2 + x_2 \\ \Rightarrow x_1 x_2^2 - x_1^2 x_2 + x_1 - x_2 = 0 \end{array} \right.$$

$$\Rightarrow x_1 x_2 (x_2 - x_1) - (x_2 - x_1) = 0 \Rightarrow (x_1 x_2 - 1)(x_2 - x_1) = 0$$

$$\Rightarrow \begin{cases} x_1 = x_2 \\ x_1 x_2 = 1 \end{cases} \quad \begin{array}{l} \text{یک به یک نیست} \\ \text{یا مساوی است} \end{array}$$

$$y = \frac{x}{x^2+1} \Rightarrow y x^2 + y = x \Rightarrow y x^2 - x + y = 0$$

$$\Rightarrow x = \frac{1 \pm \sqrt{1-4y^2}}{2y} \Rightarrow y = \frac{1 \pm \sqrt{1-4x^2}}{2x} \Rightarrow f^{-1}(x) = \frac{1 \pm \sqrt{1-4x^2}}{2x}$$

$$f(x) = \frac{x}{\omega} \Rightarrow f^{-1}\left(\frac{x}{\omega}\right) = x \Rightarrow f^{-1}(x) = \frac{1 \pm \sqrt{1-4x^2}}{2x}$$

$$f^{-1}(x) = \frac{1 - \sqrt{1-4x^2}}{2x} \xrightarrow{x = \frac{x}{\omega}} f^{-1}\left(\frac{x}{\omega}\right) = \frac{1 - \sqrt{1-4\frac{x^2}{\omega^2}}}{\frac{x}{\omega}} = \frac{\frac{x}{\omega}}{\frac{x}{\omega}} = 1 \neq x$$