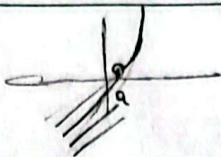


معدودی $x^3 - f$
معدودی $12x - 20$



$$\begin{array}{r} a^3 - 12a + 14 \quad | \quad a - 2 \\ a^3 - 2a^2 \\ \hline 2a^2 - 12a + 14 \\ 2a^2 - 4a \\ \hline -8a + 14 \\ a^2 + 2a - 8 = (a+4)(a-2) \end{array}$$

$$\lim_{x \rightarrow a^+} x^3 - f$$

$$\Rightarrow 12a - 20 \leq a^3 - f \Rightarrow a^3 - 12a + 14 \geq 0$$

$$\Rightarrow (a+4)(a-2)^2 \geq 0$$

$$\Rightarrow a \in [-4, +\infty)$$

$$\begin{cases} f(4) = 2 \\ f(x) = 3x + k \end{cases} \Rightarrow f(4) = 12 + k = 2 \Rightarrow k = -10 \Rightarrow \boxed{f(x) = 3x - 10}$$

الف) $f(7) = 3 \times 7 - 10 = \boxed{11}$

ب) $f(f(x)) = f(3x - 10) = 3(3x - 10) - 10 = \boxed{9x - 40}$

$$(ya, a) \in f^{-1} \Rightarrow (a, ya) \in f \Rightarrow f(a) = \frac{a^2}{a-1} = ya \Rightarrow$$

$$a^2 = ya^2 - ya \Rightarrow a^2 - ya = 0 \Rightarrow \boxed{a = 0, 2}$$

اگر $a = 0 \Rightarrow f(x) = 0$ ثابت $\Rightarrow f$ معکوس پذیر نیست

$$\Rightarrow \boxed{a = 2}$$

الف) $\{(\omega, \omega), (7, 7), (2, 2), (4, 4)\}$

$$f \circ f(x) = x$$

ب) $\{(3, 3), (4, 4), (7, 7), (9, 9)\}$

ج) $\{(2, \omega), (\omega, 4)\}$

د) $\{(3, 9), (7, 3), (9, 7)\}$

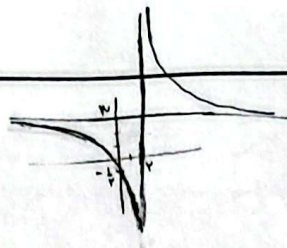
$$g^{-1} = \{(1, 2), (3, 4), (\omega, 4)\}$$

$$f \circ g^{-1} = \{(1, 4), (3, 1), (\omega, 0)\} \quad f \circ g^{-1}(x) \neq 0 \Rightarrow x \neq \omega$$

$$D_h = \{1, 3, 4, \omega\} \Rightarrow D_{\frac{h}{f \circ g^{-1}}} = \{1, 3\}$$

$$\Rightarrow \frac{h}{f \circ g^{-1}} = \{(1, \frac{1}{4}), (3, \frac{1}{1})\} = \{(1, \frac{1}{4}), (3, 1)\}$$

تابع هوموگرافیک
 جانب افقی $\frac{a}{c} = \frac{3}{1} = 3$
 جانب قائم $x=2$ = ریشه خارج
 $x=0 \Rightarrow y = -\frac{1}{3}$



$$y = \frac{3x+1}{x-2} \Rightarrow 3x - 2y = 3x + 1$$

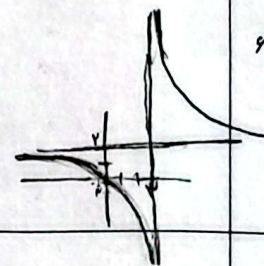
$$-2y - 1 = x(3 - y)$$

$$x = \frac{-2y-1}{3-y} = \frac{2y+1}{y-3}$$

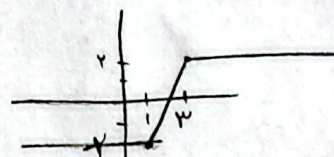
تابع معکوس

$$f^{-1}(x) = \frac{2x+1}{x-3}$$

جانب قائم $x=3$ = ریشه خارج
 جانب افقی $\frac{a}{c} = \frac{2}{1} = 2$
 $x=0 \Rightarrow y = -\frac{1}{3}$



تابع معکوس



در [1, 3] یک به یک است

$$f(x) = x-1 + x-3 = 2x-4 \quad 1 \leq x \leq 3 \Rightarrow -2 \leq y \leq 2$$

$$y + 4 = 2x \Rightarrow x = \frac{y+4}{2}$$

$$f^{-1}(y) = \frac{y+4}{2}; -2 \leq y \leq 2$$

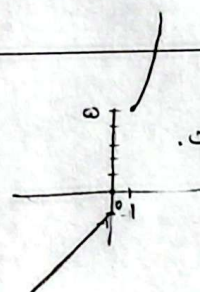
$$y = x^3 + 4 \quad x \geq 1 \Rightarrow y \geq 5$$

$$\sqrt[3]{y-4} = x \Rightarrow f^{-1}(x) = \sqrt[3]{x-4} \quad x \geq 5$$

$$y = 4x-1 \quad x \leq 0 \Rightarrow y \leq -1$$

$$\frac{y+1}{4} = x \quad f^{-1}(x) = \frac{x+1}{4} \quad x \leq -1$$

$$\Rightarrow f^{-1}(x) = \begin{cases} \sqrt[3]{x-4} & x \geq 5 \\ \frac{x+1}{4} & x \leq -1 \end{cases}$$



یک به یک و وارون پذیر است.

$$y = x^2 - \frac{(x+1)^3}{x+3} = \frac{(x^2+3x)(x+3) - (x^3+3x^2+3x+1)}{x+3} = \frac{-3x-1}{x+3}$$

$$\Rightarrow 3x + 3y = -3x - 1$$

$$3y + 1 = x(-3 - y)$$

$$x = \frac{3y+1}{-3-y} = \frac{-3y-1}{y+3} = \frac{-4y-2}{2y+6} \Rightarrow f^{-1}(x) = \frac{-4x+(-2)}{2x+4} \Rightarrow b = -2$$

$$f^{-1}(b) = f(-2) = \frac{(-3)(-2)-1}{-2+3} = \frac{5}{1} = 5$$

$$y = \frac{x}{x^2+1} \Rightarrow 3x^2 - x + y = 0 \Rightarrow x = \frac{1 \pm \sqrt{1-4y^2}}{2y} \Rightarrow f^{-1}(x) = \frac{1 \pm \sqrt{1-4x^2}}{2x}$$

$$f(3) = \frac{3}{10} \Rightarrow f^{-1}\left(\frac{3}{10}\right) = 3 = \frac{1 \pm \sqrt{1-\frac{36}{100}}}{\frac{6}{5}} = \frac{1 \pm \frac{4}{5}}{\frac{6}{5}}$$

$$\begin{aligned} & \xrightarrow{+} \frac{\frac{9}{5}}{\frac{6}{5}} = \frac{3}{2} \neq 3 \\ & \xrightarrow{-} \frac{\frac{1}{5}}{\frac{6}{5}} = \frac{1}{6} \neq 3 \end{aligned}$$

$$\Rightarrow f^{-1}(x) = \frac{1 + \sqrt{1-4x^2}}{2x}$$